

PRE-CALCULUS: by Finney, Demana, Watts and Kennedy
Geometric Sequences and Series

What you'll Learn About

- Geometric Series: Multiplying by the same

Determine if the following series is Geometric. If it is give the common ratio.

2) 3, 12, 48, 192, ...

4) 1, -2, 4, -8, ...

common ratio = $\frac{\text{next}}{\text{previous}}$

$r = -2$

$r = 4$

6) 5, 1, .2, .04, ...

10) $\frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11}, \dots$

5, 1, .2, .04, ...

Not Geometric

$r = \frac{1}{5}$

Write the first 5 terms of the geometric sequence

12) $a_1 = 4$ $r = 2$

16) $a_1 = 6$ $r = -1/4$

4, 8, 16, 32, 64

$a_1 = 6$ $r = -\frac{1}{4}$

$\frac{6}{1}, \frac{-6}{4}, \frac{6}{16}, \frac{-6}{64}, \frac{6}{256}$

18) $a_1 = 4$ $r = \sqrt{3}$

4, $4\sqrt{3}$, 12, $12\sqrt{3}$, 36

12
 $4 \cdot 3$
 $4\sqrt{9}$
 $4\sqrt{3} \cdot \sqrt{3}$

$$a_n = a_1 + \boxed{d}(n-1)$$

$$A_n = a_1 r^{n-1} \text{ when } n=1$$

or

$$A_n = a_1 r^n \text{ when } n=0$$

Use the recursive rule to write the first five terms of the sequence. Then, write the sequence as a function of n.

Explicit Formula

$$a_n = 81 \left(\frac{1}{3}\right)^{n-1}$$

$$20) a_1 = 81 \quad a_{k+1} = \frac{1}{3} a_k$$

$$a_1 = 81 \quad a_{k+1} = \frac{1}{3} a_k$$

Next term \downarrow
multiply the previous by $\frac{1}{3}$ \uparrow

$$81, 27, 9, 3, 1$$

$$a_n = a_1 (r)^{n-1}$$

$$a_2 = 5 \quad r = 3$$

$$a_n = 5(3)^{n-2}$$

$$a_0 = 5 \quad r = 3$$

$$a_n = 5(3)^{n-0}$$

$$24) a_1 = 30 \quad a_{k+1} = -\frac{2}{3} a_k$$

$$a_1 = 30 \quad a_{k+1} = -\frac{2}{3} a_k$$

$$a_n = 30 \left(-\frac{2}{3}\right)^{n-1}$$

$$\frac{30}{1}, \frac{-60}{3}, \frac{120}{9}, \frac{-240}{27}, \frac{480}{81}$$

Find the missing term of the geometric sequence

26. $a_1 = 5$ $r = \frac{3}{2}$ $n = 8$

A) $a_4 = 81$ $a_7 = 2187$ $n = 10$

$$a_n = 5 \left(\frac{3}{2}\right)^{n-1}$$

$$a_4 = 81 \quad a_7 = 2187 \quad n = 10$$

$$a_8 = 5 \left(\frac{3}{2}\right)^{8-1} \left(\frac{3}{2}\right)^9, \frac{27}{a_4}, \frac{81}{a_4}, \frac{2187}{a_7}, \dots$$

$\underbrace{\hspace{1cm}}_{x r} \quad \underbrace{\hspace{1cm}}_{x r} \quad \underbrace{\hspace{1cm}}_{x r}$

Given 2 terms

$r =$ $\times \sqrt{\text{Divide the 2 terms}}$

$x =$ Difference in term #'s

$$\frac{81}{81} r^3 = \frac{2187}{81}$$

$$r^3 = 27$$

$$r = \sqrt[3]{27}$$

$$r = 3$$

32) $a_3 = \frac{16}{3}$ $a_5 = \frac{64}{27}$ $n = 7$

(34) 3, 36, 432, ... $n = 7$

$$a_3 = \frac{16}{3}$$

$$a_5 = \frac{64}{27}$$

$$a_7 = \frac{256}{243}$$

$$a_n = 3(12)^{n-1}$$

$$r = \sqrt{\frac{\left(\frac{64}{27}\right)}{\left(\frac{16}{3}\right)}} = \frac{2}{3}$$

$$a_n = \frac{16}{3} \left(\frac{2}{3}\right)^{n-3}$$