What you'll Learn About
How to write terms given a power series
Identifying important types of power series

1. Given the series $\sum_{n=0}^{\infty} x^n$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + x^5$$

1st term = 1
$r = x$

b. Determine the function (sum) of the series (f(x) = )

$-1 < r < 1$ $\quad f(x) = \frac{a}{1-r} = \frac{1}{1-x}$

c. Substitute $x^3$ for x in the series you found in part a then simplify.

2. Given the series $\sum_{n=0}^{\infty} (-1)^n (x)^n$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty} (-1)^n (x)^n = 1 - x + x^2 - x^3 + x^4 - x^5$$

$a = 1$
$r = -x$

b. Determine the function (sum) of the series (f(x) = )

$\frac{1}{1-(-x)}$

c. Substitute $x^4$ for x in the series you found in part a then simplify.

3. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = \frac{1}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

b. Enter the first 6 terms into $y_1$ of your calculator. Use X[-π,π]$_1$ and Y[-1,1] as your window.

c. What function does it look like the series represents? That function is the sum of this series.

cos x

d. What would happen to the graphs if the first 10 terms of the series are entered into $y_1$.

e. Substitute $x^3$ for x in the series you found in part a then simplify.

$$\boxed{0! = 1}$$

$1! = 1$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

5. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$ answer the following questions.

a. List the first 6 ~~terms of the series and the general term~~

$$\underbrace{\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}}_{} = \boxed{\frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}}$$

$n=0 \quad n=1 \quad n=2$

b. Enter the first 6 terms into $y_1$ of your calculator. Use X[-π,π]$_1$ and Y[-1,1] as your window.

c. What function does it look like the series represents? That function is the sum of this series.

sin x

d. What would happen to the graphs if the first 10 terms of the series are entered into $y_1$.

It will look more and more like sine

e. Substitute $x^4$ for x in the series you found in part a then simplify.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x^4)^{2n+1}}{(2n+1)!} = \frac{x^4}{1!} - \frac{(x^4)^3}{3!} + \frac{(x^4)^5}{5!} - \frac{(x^4)^7}{7!} + \frac{(x^4)^9}{9!} - \frac{(x^4)^{11}}{11!}$$



$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{8n+4}}{(2n+1)!} = \frac{x^4}{1!} - \frac{x^{12}}{3!} + \frac{x^{20}}{5!} - \frac{x^{28}}{7!} + \frac{x^{36}}{9!} - \frac{x^{44}}{11!}$$

6. Given the series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty}\frac{x^n}{n!}=$$

b. Enter the first 6 terms into $y_1$ of your calculator. Use $X[-\pi,\pi]_1$ and $Y[-1,1]$ as your window.

c. What function does it look like the series represents? That function is the sum of this series.

d. What would happen to the graphs if the first 10 terms of the series are entered into $y_1$.

e. Substitute $x^2$ for x in the series you found in part a then simplify.

7. Given the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}=$$

b. Enter the first 6 terms into $y_1$ of your calculator. Use $X[-\pi,\pi]_1$ and $Y[-1,1]$ as your window.

c. What function does it look like the series represents? That function is the sum of this series.

d. What would happen to the graphs if the first 10 terms of the series are entered into $y_1$.

e. Substitute $x^3$ for x in the series you found in part a then simplify.

8. Given the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ answer the following questions.

a. List the first 6 terms of the series and the general term

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} =$$

b. Enter the first 6 terms into $y_1$ of your calculator. Use X[-π,π]$_1$ and Y[-1,1] as your window.

c. What function does it look like the series represents? That function is the sum of this series.

d. What would happen to the graphs if the first 10 terms of the series are entered into $y_1$.

e. Substitute $x^3$ for x in the series you found in part a then simplify.