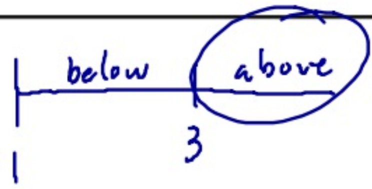
$[3, \infty)$

Solve the Inequality

very end

A) $(x-3)\sqrt{x+1} \ge 0$



$f(x) = 0$ $\quad\quad$ $\sqrt{x+1} \ge 0$

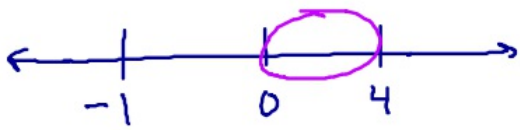
$x = 3$ $\quad\quad$ $x + 1 \ge 0$

$x \ge -1$

$f(0) = -3(1) = -3 < 0$

$f(4) = \sqrt{5} > 0$

B) $\dfrac{x^3(x-4)}{(x+1)^2} < 0$



VA: $x = -1$

zeros: $x = 0 \ / \ x = 4$

$\boxed{(0, 4)}$

$f(-2) = + / + > 0$

$f(-.5) = + / + > 0$

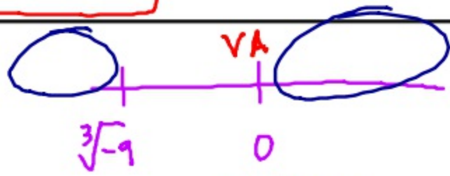
$f(1) = - / + < 0$

$f(5) = + / + > 0$

$\boxed{(-\infty, \sqrt[3]{-9}] \cup (0, \infty)}$

$\sqrt[3]{8} = 2$

C) $\frac{(x)}{(x)}\frac{x^2}{1} + \frac{9}{x} \geq 0$



$\frac{x^3}{x} + \frac{9}{x} \geq 0$

$\boxed{\frac{x^3+9}{x} \geq 0}$

$f(-3) = \frac{-27+9}{-3} > 0$

$f(-1) = 8/-1 < 0$

$f(1) = 10 > 0$

VA: $x = 0$

zeros: $x^3 + 9 = 0$

$\quad -9 \; -9$

$x^3 = -9$

$x = \sqrt[3]{-9} \approx -2.1$

D) $\frac{(x-1)}{(x-1)}\frac{5}{(x+3)} + \frac{3}{(x-1)}\frac{(x+3)}{(x+3)} < 0$

$\frac{5}{x+3} + \frac{3}{x-1} \; (-) \; 0$

$\frac{5x-5}{(x-1)(x+3)} + \frac{3x+9}{(x-1)(x+3)} < 0$

$\frac{8x+4}{(x-1)(x+3)} \; (-) \; 0$

$\boxed{\frac{8x+4}{(x-1)(x+3)} < 0}$

$8x + 4 = 0$

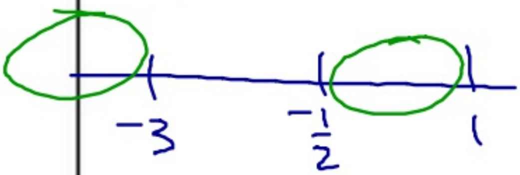
$\quad -4 \; -4$

$8x = -4$

$x = -\frac{4}{8}$

VA:

$x = 1, -3$



$\boxed{(-\infty, -3) \cup (-\frac{1}{2}, 1)}$

$f(-4) = -/+ < 0$ $\quad f(0) = 4/-3 < 0$

$f(-1) = -/- > 0$ $\quad f(2) = +/+ > 0$

Summary of the Characteristics of Graphs of Rational Functions

Horizontal Asymptote

- If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote
    - Check for an oblique/slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is y = 0

$$f(x)=\frac{3x+1}{x^2+3x+1}$$

- If the degree of the numerator and denominator are the same, the horizontal asymptote will be y = the coefficients of the highest power divided by each other

$$f(x)=\frac{5x^2+2x+1}{2x^2+3}$$

Vertical Asymptote

- Set the denominator equal to 0
- Make sure the values you get are asymptotes and not holes
- Simplify the original function by factoring
    - Whatever does not cancel is a vertical asymptote

$$f(x)=\frac{x^2+4x+3}{x+5}=\frac{(x+3)(x+1)}{(x+5)}=$$

Holes

- Set the denominator equal to zero
- Make sure the values you get are holes and not vertical asymptotes
- Simplify the original function by factoring
    - Whatever cancels is a hole

$$f(x)=\frac{x^2+4x+3}{x+1}=\frac{(x+3)(x+1)}{(x+1)}=(x+3)$$

Oblique/Slant Asymptote

- If the degree of the numerator is greater than the degree of the denominator, use long division or synthetic division to find the asymptote (y =   )
    - Ignore the remainder
    - If there is no remainder then there is not a slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

X-intercept

- Set the function = 0 (y = 0)
    - You only need to worry about the numerator in a rational function

Y-intercept

- Set the x = 0
    - The values without x will give you your y-intercept