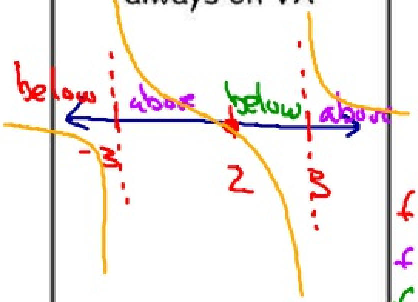
When writing intervals use brackets when necessary on zero's and parenthesis always on VA



$(x\text{-int})$ Top ←

(VA) Bottom

$(-9,-1] \cup (1,2]$

Solve the inequality

graph below x-axis

A) $\frac{x-2}{x^2-9} \le 0$

$(-\infty,-3) \cup (2,3)$

$f(x)=0$ (x-int) $x=2$

$f(x)$ und (VA)

$x^2-9=0$

$x^2=9$

$x=\pm 3$

$f(-4) = -6/7 < 0$

$f(0) = 2/9 > 0$

$f(2.5) = .5/- < 0$

$f(4) = 2/7 > 0$

B) $\frac{x^2-9}{x^2+9} \ge 0$

above

$(-\infty,-3] \cup [3,\infty)$

zeros: $x=\pm 3$

above | below | above

-3 3

$f(-4) = 7/+ > 0$

$f(0) = -1 < 0$

$f(4) = 7/+ > 0$

$x^2+9=0$

$x^2 \ne -9$

C) $\frac{x^2-x-2}{x^2+8x-9} \le 0$

$\frac{(x-2)(x+1)}{(x+9)(x-1)} \le 0$

$f(x)=0$ $x=2$ $x=-1$

$f(x)$ und $x \ne -9$ $x \ne 1$

below below

-9 -1 1 2

$f(-10) = \frac{(-)(-)}{(-)(-)} = + > 0$

$f(-2) = \frac{(-)(-)}{+(-)} < 0$

$f(0) = 2/9 > 0$

$f(1.5) = \frac{(-)(+)}{(+)(+)} < 0$

$f(3) = \frac{(+)(+)}{(+)(+)} > 0$

D) $\frac{x^3-9x}{x^2+4} > 0$

$f(x)=0$ $x(x^2-9)$

$x(x+3)(x-3)$ ✓

-3 0 3

$f(-4) = (-)(-)(-)/+ < 0$

$f(-1) = (-)(+)(-)/+ > 0$

$f(1) = ++-/+ < 0$

$f(4) = +++/+ >$

$(-3,0) \cup (3,\infty)$

	Solve the Inequality A) $(x-3)\sqrt{x+1} \geq 0$ B) $\frac{x^3(x-4)}{(x+1)^2} < 0$

	C) $x^2 + \frac{9}{x} \geq 0$ D) $\frac{5}{x+3} + \frac{3}{x-1} < 0$

Summary of the Characteristics of Graphs of Rational Functions

Horizontal Asymptote

- If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote
  - Check for an oblique/slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is y = 0

$$f(x)=\frac{3x+1}{x^2+3x+1}$$

- If the degree of the numerator and denominator are the same, the horizontal asymptote will be y = the coefficients of the highest power divided by each other

$$f(x)=\frac{5x^2+2x+1}{2x^2+3}$$

Vertical Asymptote

- Set the denominator equal to 0
- Make sure the values you get are asymptotes and not holes
- Simplify the original function by factoring
  - Whatever does not cancel is a vertical asymptote

$$f(x)=\frac{x^2+4x+3}{x+5}=\frac{(x+3)(x+1)}{(x+5)}=$$

Holes

- Set the denominator equal to zero
- Make sure the values you get are holes and not vertical asymptotes
- Simplify the original function by factoring
  - Whatever cancels is a hole

$$f(x)=\frac{x^2+4x+3}{x+1}=\frac{(x+3)(x+1)}{(x+1)}=(x+3)$$

Oblique/Slant Asymptote

- If the degree of the numerator is greater than the degree of the denominator, use long division or synthetic division to find the asymptote (y =   )
  - Ignore the remainder
  - If there is no remainder then there is not a slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

X-intercept

- Set the function = 0 (y = 0)
  - You only need to worry about the numerator in a rational function

Y-intercept

- Set the x = 0
  - The values without x will give you your y-intercept