

Find the polynomial function with leading coefficient 4 that has the given degree and zeros.

A) Degree 3, with 2, -1, and 4 as zeros  $\rightarrow x^2 + 1x - 2x - 2$

$$f(x) = 4(x-2)(x+1)(x-4) \rightarrow (4x-8)(x-2)(x-4)$$

$$f(x) = 4[(x^2 - 1x - 2)(x-4)]$$

$$= 4[x^3 - x^2 - 2x - 4x^2 + 4x + 8]$$

$$= 4[x^3 - 5x^2 + 2x + 8]$$

$$= 4x^3 - 20x^2 + 8x + 32$$

Leading coefficient of 4

B) Degree 3 with 5, 1/3, and 2/3 as zeros

$$\begin{array}{l} x=5 \quad (3) \quad x = \frac{1}{3} \quad x = \frac{2}{3} \\ \underline{-5 \quad -5} \quad \underline{3x = 1} \\ x-5=0 \quad \underline{-1 \quad -1} \\ (x-5) \quad (3x-1)=0 \end{array}$$

$$f(x) = \frac{4}{9}(x-5)(3x-1)(3x-2)$$

$$x(3x)(3x)$$

$$9x^3$$

$$\frac{9(\text{what})}{9} = \frac{4}{9}$$

Using only algebraic methods, find the cubic function with the given table of values

x	-4	0	3	5
f(x)	0	180	0	0

x-int

x-int

x-int

y-int

y-int

$$(4)(-3)(-5) = 60$$

$$60(\text{what}) = 180$$

$$y = a(x+4)(x-3)(x-5)$$

$$180 = a(0+4)(0-3)(0-5)$$

$$\frac{180}{60} = \frac{60a}{60} \quad a = 3$$

$$y = 3(x+4)(x-3)(x-5)$$

Finding Rational Zeros

- 1) List all possible rational zeros p/q where q is the leading coefficient and p is the constant
- 2) Use your calculator to find the zeros and then use synthetic division and algebra to prove that the zeros that you chose are rational zeros

$x = 2, -2, \frac{2}{3}, -\frac{2}{3}$   
 $\boxed{1} \times \frac{1}{3}, -\frac{1}{3}$

Rational #'s

$p = 1$   
 $q = 1$   
 $\frac{p}{q} = \pm 1$

Use the Rational Zeros Theorem to write a list of all potential rational zeros and then determine which ones, if any, are zeros.

$f(x) = 3x^3 + 4x^2 - 5x - 2$

$p = -2$      $q = 3$   
 $\begin{matrix} \diagup & \diagdown \\ 2 & 1 \\ -2 & -1 \end{matrix}$      $\begin{matrix} \diagup & \diagdown \\ 1 & 3 \\ -1 & -3 \end{matrix}$

$\frac{p}{q} = \pm \frac{2}{1}, \pm \frac{2}{3}, \pm \frac{1}{1}, \pm \frac{1}{3}$  Possible zeros

$\begin{array}{r|rrrr} +1 & 3 & 4 & -5 & -2 \\ & & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & 0 \end{array}$      $x = 1$

$3x^2 + 7x + 2 = 0$

$(3x + 1)(x + 2) = 0$

$x = -\frac{1}{3}$      $x = -2$

Irrational  
- square roots

Use the Rational Zeros Theorem to write a list of all potential rational zeros and then determine which ones, if any, are zeros.

$f(x) = x^3 - 3x^2 + 1$

$\begin{array}{r|rrrr} -1 & 1 & -3 & 0 & 1 \\ & & -1 & +1 & -1 \\ \hline & 1 & -4 & 1 & 0 \end{array}$

No Rational Zeros

Find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

52.  $f(x) = x^3 - 6x^2 + 7x + 4$

$$\begin{array}{r|rrrrr} 4 & 1 & -6 & 7 & 4 & \\ & & 4 & -8 & -4 & \\ \hline & 1 & -2 & -1 & 0 & \end{array}$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2} = \boxed{\frac{1 \pm \sqrt{8}}{2}} = 1 \pm \sqrt{4}$$

Find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

$f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

$$\begin{array}{r|rrrrrr} 4 & 2 & -7 & -8 & 14 & 8 & \\ & & 8 & 4 & -16 & -8 & \\ \hline & 2 & 1 & -4 & -2 & 0 & \\ -\frac{1}{2} & & & -1 & 0 & 2 & \\ \hline & 2 & 0 & -4 & 0 & \end{array}$$

$$\begin{aligned} 2x^2 - 4 &= 0 \\ 2x^2 &= 4 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

$$\begin{aligned} x &= \sqrt{2}, -\sqrt{2} \text{ Irrational} \\ x &= 4, -\frac{1}{2} \text{ Rational} \end{aligned}$$