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What you'll Learn About

$f^{-1}(x) \to f$ inverse

① Switch x and y
② Resolve for y

Reflection over the line $y=x$
$y=f(x)$ $y=f^{-1}(x)$

Find a formula for $f^{-1}(x)$. Give the domain of $f^{-1}(x)$, including any restrictions "inherited" from f.

A. $f(x)=\sqrt{x}$

$y=\sqrt{x}$ D: $x\geq 0$ R: $y\geq 0$

$(x)^2=(\sqrt{y})^2$

$x^2=y$

$\boxed{f^{-1}(x)=x^2}$

D: $x\geq 0$
R: $y\geq 0$

B. $f(x)=\sqrt[3]{2x+1}$

$y=\sqrt[3]{2x+1}$

D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

$(x)^3=(\sqrt[3]{2y+1})^3$

$x^3=2y+1$
$-1 \qquad -1$

$\frac{x^3-1}{2}=\frac{2y}{2}$

$f^{-1}(x)=\frac{x^3-1}{2}$

D: $(-\infty,\infty)$

C. $f(x)=5x+2$

$y=5x+2$ D: $(-\infty,\infty)$ R: $(-\infty,\infty)$

$x=5y+2$
$-2 \qquad -2$

$\frac{x-2}{5}=\frac{5y}{5}$

$\frac{x-2}{5}=y$

$\boxed{f^{-1}(x)=\frac{x-2}{5}}$ $\boxed{D: (-\infty,\infty)}$

D. $f(x)=\sqrt{x^3-2}$

$y=\sqrt{x^3-2}$ D: $x^3-2\geq 0$
$x^3\geq 2$
$x\geq\sqrt[3]{2}$

$(x)^2=(\sqrt{y^3-2})^2$ R: $y\geq 0$

$x^2=y^3-2$

$x^2+2=y^3$

$\boxed{\sqrt[3]{x^2+2}=f^{-1}(x)}$ D: $x\geq 0$

(E) $f(x) = \frac{3x+2}{x-1}$

D: $x \neq 1$ $(-\infty, 1) \cup (1, \infty)$

R: HA: $y = 3$ $(-\infty, 3) \cup (3, \infty)$

$y = \frac{3x+2}{x-1}$

$(y-1)\, x = \frac{3y+2}{\cancel{y-1}} \cancel{(y-1)}$

$xy - x = 3y + 2$
$\quad -2 \qquad\qquad -2$

$xy - x - 2 = 3y$
$-xy \qquad\qquad -xy$

$-x - 2 = 3y - xy$

$\frac{-x-2}{(3-x)} = \frac{y(3-x)}{(3-x)}$ → GCF

$\boxed{\frac{-x-2}{3-x} = f^{-1}(x) \quad D: (-\infty, 3) \cup (3, \infty)}$

F $f(x) = x^2 + 3$

D: $(-\infty, \infty)$

R: $[3, \infty)$

$f^{-1}(x) = \sqrt{x-3}$

D: $[3, \infty)$

R: $\cancel{(-\infty, \infty)} \rightarrow [0, \infty)$