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Domain/Range

HA / VA / HOLE

$f(-5) = \frac{3(-5)^2+15(-5)}{-5+5} = \frac{75-75}{-5+-} = \frac{0}{0}$

Determine if each function is continuous. If the function is not continuous, find the x-axis location of each discontinuity and classify each discontinuity as infinite or removable. Also find any horizontal asymptotes.

A) $f(x) = \frac{3x^2+15x}{x+5} = \frac{3x\cancel{(x+5)}}{\cancel{(x+5)}} = \boxed{3x}$ B) $f(x) = \frac{x^2+3x}{x+2}$

Simplified expression

Hole

~~VA~~ $x+5=0$

Hole $x=-5$

D: $(-\infty,-5)\cup(-5,\infty)$

R: $(-\infty,-15)\cup(-15,\infty)$

HA: NONE

To find the y-value of hole Plug x-value into simplified expression

C) $f(x) = \frac{9x+6}{x^2-4}$ D) $f(x) = \frac{9x+18}{x^2-4}$

$f(x) = \frac{\cancel{x-5}}{\cancel{(x-5)}(x+1)} = \frac{1}{x+1}$ $f(5) = \frac{1}{6}$ ← Hole

E) $f(x) = \frac{x-5}{x^2-4x-5}$

HA: $y=0$

VA: $x^2-4x-5=0$

$(x-5)(x+1)=0$

D: $(-\infty,-1)\cup(-1,5)\cup(5,\infty)$

R: $(-\infty,0)\cup(0,\frac{1}{6})\cup(\frac{1}{6},\infty)$

$x=5$ Hole $x=-1$ VA

Summary of the Characteristics of Graphs of Rational Functions

Horizontal Asymptote

- If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote

  $$f(x)=\frac{x^2+2x+1}{x+3}$$

- If the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is y = 0

  $$f(x)=\frac{3x+1}{x^2+3x+1}$$

- If the degree of the numerator and denominator are the same, the horizontal asymptote will be y = the coefficients of the highest power divided by each other

  $$f(x)=\frac{5x^2+2x+1}{2x^2+3}$$

Vertical Asymptote (non-removeable discontinuities)

- Set the denominator equal to 0
- Make sure the values you get are asymptotes and not holes
- Substitute the value back into the function.
- $\frac{a}{0}$, where is any number but 0, then there is a vertical asymptote

Holes (removeable discontinuities)

- Set the denominator equal to zero
- Make sure the values you get are holes and not vertical asymptotes
- Substitute the value back into the function.
    - $\frac{0}{0}$ is a hole
- To find where the hole is
    - Simplify the original function by factoring
    - Substitute the value from the domain into the cancelled function to find the y-value of the hole

$$f(x)=\frac{x^2+4x+3}{x+1}=\frac{(x+3)(x+1)}{(x+1)}=(x+3)$$

Oblique/Slant Asymptote

- If the degree of the numerator is greater than the degree of the denominator, use long division or synthetic division to find the asymptote (y =   )
    - Ignore the remainder
    - If there is no remainder then there is not a slant asymptote

$$f(x)=\frac{x^2+2x+1}{x+3}$$

X-intercept

- Set the function = 0 (y = 0)
  - You only need to worry about the numerator in a rational function

Y-intercept

- Set the x = 0
  - The values without x will give you your y-intercept

Even/Odd/Neither

- Plug a number to the left of x = 0 into the function and a number to the right of x = 0 into the function (x = -1 and x = 1 usually work.
  - If f(-1) = f(1) the function is even
    - Even functions also reflect over the y-axis
  - If f(-1) is the opposite of f(1) then the function is odd
    - Odd functions have rotational symmetry about the origin
  - If the f(-1) and f(1) are not the same or opposites the function is neither even or odd

Domain:

- If you have a fraction, set the denominator = 0
- If you have a square root in the numerator, set what is under the square root greater than or equal to 0
- If there is a square root in the denominator, set what is under the square roo greater than zero.

Range:

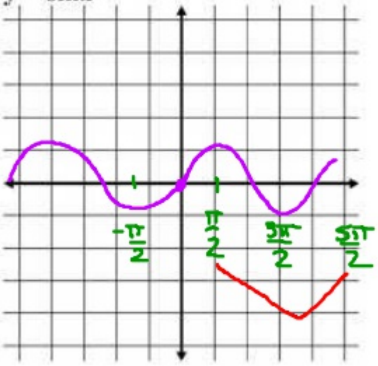
- Look at the graph for complicated functions, but be familiar with the ranges for the function given below

The domain and range of a linear/cubic function are always $(-\infty,\infty)$.

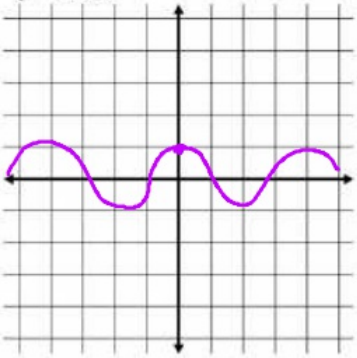
The domain of a quadratic function is always $(-\infty,\infty)$.
The range of a quadratic function that opens up is $[\text{y valueof themin},\infty)$.
The range of a quadratic function that opens down is $(-\infty, \text{y valueof themax}]$

The domain of sine and cosine are always $(-\infty,\infty)$.
The range of sine of and cosine are always [minimum y-value, maximum y-value].

y = sinx



y = cosx



1) Determine the domain and range

D: $(-\infty, \infty)$

R: $[-1, 1]$

1) Determine the domain and range

2) Is the function even, odd or neither

odd

2) Is the function even, odd or neither

even

3) Intervals of Increase or Decrease

3) Intervals of Increase or Decrease

4) Find any extrema.

max $x = \frac{\pi}{2} + 2\pi k$

where k is any integer

min $x = -\frac{\pi}{2} + 2\pi k$

4) Find any extrema.

5) Determine the end behavior

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6) Find any asymptotes

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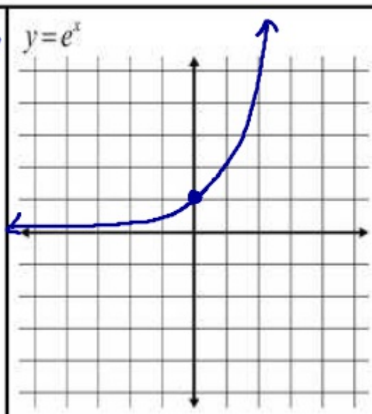
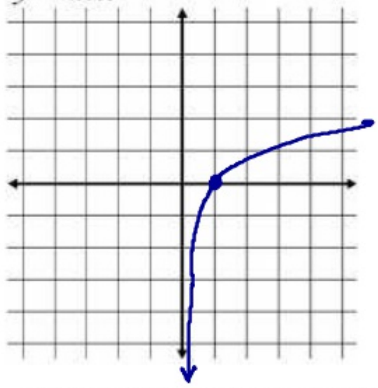
natural logarithm

exponential → $y=e^x$

$e = 2.718....$



$y = \ln x$



$y=e^x$	$y = \ln x$
1) Determine the domain and range	1) Determine the domain and range
2) Is the function even, odd or neither	2) Is the function even, odd or neither
3) Intervals of Increase or Decrease	3) Intervals of Increase or Decrease
4) Find any extrema.	4) Find any extrema.
5) Determine the end behavior	5) Determine the end behavior
6) Find any asymptotes	6) Find any asymptotes