Semester Review Topics

15. $\lim_{x \to 0^+} \dfrac{1 + \sin(x)}{x}$ is

$\dfrac{1 + \sin(0)}{0} = \dfrac{1}{0}$

(A) 0

(B) 1

(C) 2

(D) π

(E) ∞

$$\lim_{x \to 0} \frac{\sin(5x)}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{5\cos(5x)}{1} = \frac{5\cos(0)}{1} = 5$$

Tangent Line

Find the equation of the line tangent to the curve $f(x) = x^3 - 4x$ at $x = 0$.

$f'(x) = 3x^2 - 4$

$(0, 0)$

$f'(0) = -4$

$\boxed{y = 0 - 4(x - 0)}$

Increasing Functions

$(-\infty, -\sqrt{\tfrac{4}{3}})$

$(\sqrt{\tfrac{4}{3}}, \infty)$

What are all values of x for which the function f defined by $f(x) = x^3 - 4x$ are increasing

$$\boxed{f'(x) = 3x^2 - 4}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

$-\sqrt{\tfrac{4}{3}}$ $\quad$ $\sqrt{\tfrac{4}{3}}$

$$0 = 3x^2 - 4$$

$$\frac{4}{3} = \frac{3x^2}{3}$$

$$\frac{4}{3} = x^2$$

$f'(-2) = 8 > 0$ f inc

$f'(0) = -4 < 0$ f dec

$f'(2) = 8 > 0$ f inc

Inflection Points

Determine any points of inflection for the curve $f(x) = x^3 - 4x$

$f'(x) = 3x^2 - 4$

$\boxed{f''(x) = 6x}$

$0 = 6x$

$\frac{0}{6} = \frac{6x}{6}$

$0 = x$

$f''(-1) = -6 < 0$ $f(x)$ concave down

$f''(1) = 6 > 0$ $f(x)$ concave up

$x = 0$ is a pt. of inflection because the sign of f'' changes

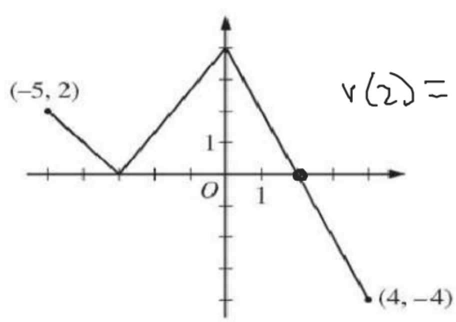
Let f be the differentiable function whose graph is shown in the figure. The position, in meters, at time t (sec) of a particle moving along a horizontal coordinate axis is given by $s(t)=\int_0^x f(t)dt$ Use the graph of f(x) below to answer the questions.

a. Find the velocity of the particle at t = 2.

$s(t) =$ area

$v(t) = f(x)$ graph

$a(t) = f'(x)$ slopes of graph

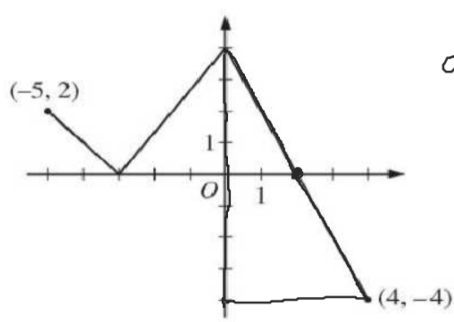


Graph of f

$v(2) = 0$ m/sec

Let f be the differentiable function whose graph is shown in the figure. The position, in meters, at time t (sec) of a particle moving along a horizontal coordinate axis is given by $s(t)=\int_0^x f(t)dt$ Use the graph of f(x) below to answer the questions.

b. Find the acceleration of the particle at t = 2.

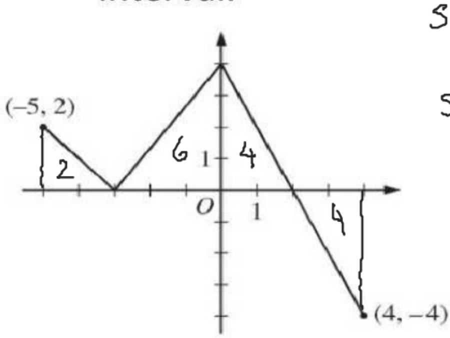


Graph of f

$a(2) = -\frac{8}{4} = -2\ m/sec^2$

Let f be the differentiable function whose graph is shown in the figure. The position, in meters, at time t (sec) ~~of a particle~~ moving along a horizontal coordinate axis is given by $s(t) = \int_0^x f(t)dt$ Use the graph of f(x) below to answer the questions.

c. Find the absolute maximum and minimum of s(t) on the given interval.



Graph of f

$s(-5) = \int_0^{-5} f(t) = -8$ → Abs min $x = -5$

$s(-3) = \int_0^{-3} f(t) = -6$

$s(2) = \int_0^{2} f(t) = 4$ → Abs max $x = 2$

$s(4) = \int_0^{4} f(t) = 0$

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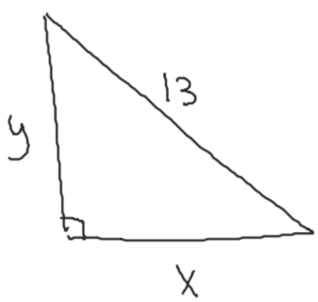
Related Rates

A 13 ft ladder is leaning against a house when it's base starts to slide away. By the time the base is 12 feet $\rightarrow x$ from the house the base is moving at a rate of 5 ft/sec. $\rightarrow \frac{dx}{dt}$

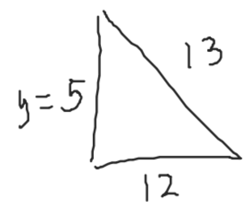
$\frac{dy}{dt}$ ~~a)~~ How fast is the top of the ladder sliding down the wall at that moment?



$$x^2+y^2=13^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$2(12)(5)+2(5)\frac{dy}{dt}=0$$



$$120+10\frac{dy}{dt}=0$$

$$10\frac{dy}{dt}=-120$$

$dy/dt=-12$ ft/sec

$$A = \frac{1}{2}h(b_1 + b_2)$$

Use the data below to approximate the area under the curve using the Trapezoid Rule with 4 sub-intervals.

t	0	2	5	9	10
H(t)	66	60	52	44	43

$$A = \frac{1}{2}(2-0)(66+60) + \frac{1}{2}(5-2)(60+52) + \frac{1}{2}(9-5)(52+44) + \frac{1}{2}(10-9)(44+43)$$

$$A = \frac{1}{2}(2)(126) + \frac{1}{2}(3)(112) + \frac{1}{2}(4)(96) + \frac{1}{2}(1)(87)$$

$$A = 126 + 168 + 192 + 43.5 = 529.5$$

$A = bh$

Use the data below to approximate the area under the curve using a Right Riemann Sum with 4 sub-intervals.

t	0	2	5	9	10
H(t)	66	60	52	44	43

$A = (2-0)(60) + (5-2)(52) + (9-5)(44) + (10-9)(43)$

$A = 120 + 156 + 176 + 43$

495

Use the data below to approximate the area under the curve using a Left Riemann Sum with 4 sub-intervals.

t	0	2	5	9	10
H(t)	66	60	52	44	43

$A = 2(66) + 3(60) + 4(52) + 1(44)$

564

- Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, from [0, 6], is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.5

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	**0**	**5.3**	**8.8**	**11.2**	**12.8**	**13.8**	**14.5**

Avg

- Use a midpoint sum with three subinterval of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)dt.$

$\frac{1}{6}\left[(2-0)(5.3)+(4-2)(11.2)+(6-4)(13.8)\right]$

$\frac{1}{6}\left[10.6+22.4+27.6\right]=10.1$ ounces

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.5

- Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)dt$ in the context of the problem.

Average amount of amount of water dripping through the coffee pot during the first 6 seconds was 10.1 ounces.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.5

- Find the value of $C'(4.5)$

$$C'(4.5) = \frac{13.8-12.8}{5-4} = 1 \text{ ounces/min}$$

- Using correct units, explain the meaning of $C'(4.5)$ in the context of the problem.

The rate that coffee is dripping into the pot at $t=4.5$ min is 1 ounce per min.