Test Chapter 9 (9.1–9.3 Practice Test) Name:______________________________

$a = x^4$

$r = -x^3$

1) Find a power series representation for $\frac{x^4}{1+x^3}$. (Include a formula for the nth term and a formula for the summation). Then find the interval of convergence for the series.

$$\frac{x^4}{1+x^3} = x^4 - x^7 + x^{10} + \cdots (-1)^n x^4 (x^{3n}) + \cdots \sum_{n=0}^{\infty} (-1)^n x^{3n+4}$$

$$-1 < x^3 < 1$$

$$\boxed{-1 < x < 1}$$

2) Find the first three nonzero terms for the Taylor Polynomial generated by $f(x) = \cos 2x$ at $x = \frac{\pi}{2}$.

$$P_4(x) = -1 + \frac{4\left(x-\frac{\pi}{2}\right)^2}{2} - \frac{16\left(x-\frac{\pi}{2}\right)^4}{4!}$$

$$= -1 + 2\left(x-\frac{\pi}{2}\right)^2 - \frac{2\left(x-\frac{\pi}{2}\right)^4}{3}$$

$f(x) = \cos(2x) \to -1$

$f'(x) = -2\sin(2x) \to 0$

$f''(x) = -4\cos(2x) \to 4$

$f'''(x) = +8\sin(2x) \to 0$

$f^4(x) = 16\cos(2x) \to -16$

$f^5(x) = -32\sin(2x) \to 0$

$f^6(x) = -64(\cos(2x)) = 64$

B) Show that $|f(x)-P_4(x)| < \frac{1}{2000000000}$ between $\frac{23\pi}{45} < x < \frac{\pi}{2}$

$$\frac{64(x-2)^6}{6!} = \frac{64\left(\frac{23\pi}{45}-\frac{\pi}{2}\right)^6}{6!} < \frac{1}{2000000000}$$

C) Find $\left|f\left(\frac{23\pi}{45}\right)-P_4\left(\frac{23\pi}{45}\right)\right| = 1.608 \times 10^{-10}$

3) a) Find the Taylor polynomial of order 10 for $f(x) = \ln(1+x^5)$ at $x = 0$.

$$x^5 - \frac{x^{10}}{2}$$

b. Show that $|f(x)-P_{10}(x)| \le 5 \times 10^{-20}$ between $0 < x < .05$

$$\frac{x^{15}}{3} = \frac{.05^{15}}{3} \le 5 \times 10^{-20}$$

4) a) Find the Taylor polynomial of order 2 for $f(x) = \tan^{-1}(x^2)$ at $x = 0$.

$$P_2(x) = x^2$$

B) Show that $|f(x) - P_2(x)| < \frac{3}{100000}$ between $0 < x < .2$

$$\frac{x^6}{3} = \frac{.2^6}{3} < \frac{3}{100000}$$

C) Find $|f(.2) - P_2(.2)|$ $.0000213$

5) Let f be a function that has derivatives of all orders on the interval $(-\infty, \infty)$. Assume that $f(3) = -2$, $f'(3) = 5$, $f''(3) = -21$, and $f'''(3) = 40$.

(a) Find the third-order Taylor polynomial about $x = 3$ for $f(x)$.

$$P_3(x-3) = -2 + 5(x-3) - \frac{21(x-3)^2}{2} + \frac{40(x-3)^3}{6}$$

(b) Find the second-order Taylor polynomial about $x = 3$ for $f'(x)$.

$$P_2(x-3) = -\cancel{2} + 5 - 21(x-3) + \frac{120(x-3)^2}{6}$$

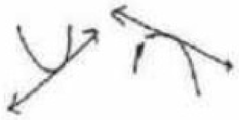
(c) Find the fourth-order Taylor polynomial about $x = 3$ for

$$\int_0^x f(t)\,dt$$

$$P_4(x-3) = -\cancel{2x} + \frac{5(x-3)^2}{2} - \frac{21(x-3)^3}{6} + \frac{40(x-3)^4}{24}$$

$$-2(x-3)$$

(d) ~~Find the ... Taylor polynomial ...~~ Determine if the linearization of f is an over or underestimate of f near 3.



$f''(3) = -21 < 0$ f is concave down

the linearization is an overestimate

6) Use a familiar MacClaurin series to help you integrate the following. Make sure you include a rule for the nth term and a rule for the sum.

$$\int \frac{\ln(1+t^2)}{t} = \int \frac{t^2 - \frac{t^4}{2} + \frac{t^6}{3} - \cdots \frac{(-1)^{n-1} t^{2n}}{n} + \cdots \sum_{n=0}^{\infty} \frac{(-1)^{n-1} t^{2n}}{n}}{t}$$

$$\int t - \frac{t^3}{2} + \frac{t^5}{3} - \cdots \frac{(-1)^{n-1} t^{2n-1}}{n} + \cdots \sum \frac{(-1)^{n-1} t^{2n-1}}{n}$$

$$\frac{1}{2}t^2 - \frac{t^4}{8} + \frac{t^6}{18} - \cdots \frac{(-1)^{n-1} t^{2n}}{n(2n)} + \cdots \sum_{n=1}^{\infty} \frac{(-1)^{n-1} t^{2n}}{n(2n)}$$

7) Use a familiar MacClaurin series to help you integrate the following. Make sure you include a rule for the nth term and a rule for the sum.

$$\int \tan^{-1}(x^4) = \quad x^4 - \frac{x^{12}}{3} + \frac{x^{20}}{4} + \cdots \frac{(-1)^n x^{8n+4}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{8n+4}}{2n+1}$$

$$\int \tan^{-1}(x^4) = \frac{1}{5}x^5 - \frac{1}{39}x^{13} + \frac{x^{21}}{105} + \cdots \frac{(-1)^n x^{8n+5}}{(2n+1)(8n+5)} + \cdots \sum_{n=0}^{\infty} \frac{(-1)^n x^{8n+5}}{(8n+5)(2n+1)}$$

8) Use a familiar MacClaurin series to help you integrate the following. Make sure you include a rule for the nth term and a rule for the sum.

$$\longrightarrow \int \frac{x^4}{1-x^2} = \int x^4 + x^6 + x^8 + \cdots x^4(x^{2n}) + \cdots \sum_{n=0}^{\infty} x^{2n+4}$$

$$= \frac{1}{5}x^5 + \frac{1}{7}x^7 + \frac{1}{9}x^9 + \cdots \frac{x^{2n+5}}{2n+5} + \cdots \sum_{n=0}^{\infty} \frac{x^{2n+5}}{2n+5}$$

$$\frac{d}{dx}\left[\frac{x^4}{1-x^2}\right] = 4x^3 + 6x^5 + 8x^7 + \cdots (2n+4)x^{2n+3} + \cdots$$