The Taylor series for a function f about x = 1 is given by $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{4^n}{n}(x-1)^n$ and converges to f(x) for $|x-1|<R$, where r is the radius of convergence of the Taylor series.

$-1 < 4x-4 < 1$

$3 < 4x < 5$

$\boxed{\frac{3}{4} < x \leq \frac{5}{4}}$

a) Find the interval of convergence for Taylor Series of f.

$$\lim_{n\to\infty}\left|\frac{4^{n+1}(x-1)^{n+1}}{n+1}\cdot\frac{n}{4^n(x-1)^n}\right| = |4(x-1)| < 1$$

$x=\frac{5}{4}$ $\sum\frac{(-1)^{n+1}4^n\left(\frac{1}{4}\right)^n}{n} = \sum\frac{(-1)^{n+1}}{n}$

$\lim_{n\to\infty}\frac{1}{n}=0$

$\frac{1}{n+1}\leq\frac{1}{n}$

$x=\frac{3}{4}$ $\sum\frac{(-1)^{n+1}4^n\left(-\frac{1}{4}\right)^n}{n}$

$\sum\frac{(-1)^{n+1}(-1)^n}{n} = \sum\frac{(-1)^{2n+1}}{n}$ Harmonic diverges

Abs.? $\sum\frac{1}{n}$ Harmonic Diverges

b) Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about x = 1.

$$\frac{d}{dx}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}4^n}{n}(x-1)^n = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}4^n}{\cancel{n}}\cancel{n}(x-1)^{n-1} = \sum_{n=1}^{\infty}(-1)^{n+1}4^n(x-1)^{n-1}$$

$$= 4 - 16(x-1) + 64(x-1)^2$$

c) Show that $|P_2(1.1)-f'(1.1)|\leq\frac{32}{125}$.

$|P_2(1.1)-f'(1.1)| \leq 256(x-1)^3$

$\leq 256(1.1-1)^3$

$\leq .256 = \frac{32}{125}$

Since part b is an alternating series, the error comes from the next term.

d) The Taylor series for f' about x = 1, found in part (b), is a geometric series. Find the **function** f' to which the series converges for $|x-1|<R$

$$f'(x) = \frac{a}{1-r} = \frac{4}{1+4(x-1)} = \frac{4}{1+4x-4} = \frac{4}{4x-3}$$

$a=4$

$r=-4(x-1)$

Centered at 1

e) Use the function in part d to determine f for $|x-1|<R$.

$f'(x)=\frac{4}{4x-3}$ $\quad f=\int\frac{4}{4x-3} = \ln|4x-3|+C$

Determine the value of $\sum_{n=0}^{\infty}(-1)^n \frac{\left(\frac{1}{2}\right)^{2n+1}}{(2n+1)!}$ $= \sin\left(\frac{1}{2}\right)$

Find the
Sum

Determine the value of the following series given below

$$1-\frac{\left(\frac{\pi}{4}\right)^2}{2!}+\frac{\left(\frac{\pi}{4}\right)^2}{4!}-\cdots(-1)^n\frac{\left(\frac{\pi}{4}\right)^{2n}}{(2n)!} = \cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

Determine $f^7(x)$ in the series

$$x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}\cdots+(-1)^n\frac{x^{2n+1}}{2n+1}$$

$$\frac{f^7(0)\,x^7}{7!}=\frac{x^7}{7} \qquad f^7(0)=\frac{7!}{7}$$

$$\frac{f^7(0)}{7!}=\frac{1}{7} \qquad \boxed{f^7(0)=6!}$$