Test Chapter 9 (9.4-9.5 Practice Test) Name:______________________

Not Use Calculator

1) Which of the following series converges?

A) ~~$\sum_{n=1}^{\infty} \frac{1}{n}$~~ Harmonic

B) ~~$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$~~ $p = 1/2$

C) $\sum_{n=1}^{\infty} n^{-5} = \sum \frac{1}{n^5}$ $p = 5 > 1$

D) ~~$\sum_{n=1}^{\infty} n^{-3/4}$~~ $= \sum \frac{1}{n^{3/4}}$ $p = 3/4 < 1$

2) Describe the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

$\sum \frac{1}{n^2}$ $p = 2 > 1$

$\lim_{n \to \infty} \frac{1}{n^2} = 0$

$\left(\frac{1}{n+1}\right)^2 < \frac{1}{n^2}$

~~A) Diverges~~

B) Converges absolutely

C) Converges conditionally

~~D) Cannot be determined.~~

3) Does the following series converge or diverge? (Show the test that leads to your conclusion)

$$\sum_{n=1}^{\infty} \frac{n^3}{4^n}$$

$$\lim_{n \to \infty} \left| \frac{(n+1)^3}{4^{n+1}} \cdot \frac{4^n}{n^3} \right| = \left| \frac{(n+1)^3}{4n^3} \right| = \frac{1}{4} < 1$$

absolute convergence

4) Determine if the following series converges absolutely, converges conditionally, or diverges?

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{5/4}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{5/4}} \rightarrow \text{Converges Absolutely}$$

Conditional since,

The absolute value of the terms approach zero and the next term is smaller than the previous

$\sum \frac{1}{n^{5/4}}$ $p = \frac{5}{4} > 1$ converge

5) Determine if the following series converges absolutely, conditionally, or diverges.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{9^n}$$

$r = -\frac{1}{9}$

$|r| < 1$

converges absolutely

6) Does the following series converge or diverge? Show the test that leads to your conclusion.

$$\sum_{n=1}^{\infty} \frac{(6-5n)^n}{(2n)^n}$$ → the sum diverges

$$\lim_{n\to\infty} \left| \left[\left(\frac{6-5n}{2n} \right)^n \right]^{1/n} \right| = \left| \frac{6-5n}{2n} \right| = \frac{5}{2} > 1$$

7) Determine whether the series $\boxed{\sum_{n=1}^{\infty} \frac{n^3}{(5n-1)!}}$ converges or diverges. Identify the test (or tests) you used. → Converges absolutely

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(5(n+1)-1)!} \cdot \frac{(5n-1)!}{n^3} \right| = \left| \frac{(n+1)^3 (5n-1)!}{n^3 (5n+4)!} \right| = \left| \frac{(n+1)^3}{n^3 (5n+4)(5n+3)(5n+2)(5n+1)(5n)} \right|$$

$0 < 1$

8) Find the radius of convergence for $\sum_{n=0}^{\infty} \frac{(6x-4)^n}{n!}$.

Explain your answer.

$$\lim_{n\to\infty} \left| \frac{(6x-4)^{n+1}}{(n+1)!} \cdot \frac{n!}{(6x-4)^n} \right| = \left| \frac{6x-4}{n+1} \right| = 0 < 1$$

converges always

$R.O.C = \infty$

9) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(3x-1)^{3n}}{2^n}$ and, within this interval, the sum of the series as a function of x.

$$r = \frac{(3x-1)^3}{2}$$

$$S = f(x) = \frac{1}{1 - \frac{(3x-1)^3}{2}} = \frac{2}{2-(3x-1)^3}$$

$$-1 < \frac{(3x-1)^3}{2} < 1$$

$$-2 < (3x-1)^3 < 2$$

$$\sqrt[3]{-2} < 3x-1 < \sqrt[3]{2}$$

$$1 + \sqrt[3]{-2} < 3x < 1 + \sqrt[3]{2}$$

$$\frac{1+\sqrt[3]{-2}}{3} < x < \frac{1+\sqrt[3]{2}}{3}$$

10) Find the interval of convergence and the radius of convergence for $\sum_{n=0}^{\infty} \frac{n^3(2x-5)^n}{2^n}$.

Show the tests that lead to your conclusion.

$$\lim_{n\to\infty}\left|\frac{(n+1)^3(2x-5)^{n+1}}{2^{n+1}}\cdot\frac{2^n}{n^3(2x-5)^n}\right| = \left|\frac{(n+1)^3(2x-5)}{2n^3}\right| = \left|\frac{2x-5}{2}\right| < 1$$

$$-1 < \frac{2x-5}{2} < 1$$

$$-2 < 2x-5 < 2$$

$$3 < 2x < 7$$

$$\boxed{\frac{3}{2} < x < \frac{7}{2}} \quad \text{I.O.C}$$

$$1 = \text{R.O.C}$$

$x = \frac{3}{2}$ $\quad \sum \frac{n^3(3-5)^n}{2^n} = \sum \frac{n^3(-2)^n}{2^n} = \sum (-1)^n n^3$

Diverges

$\lim_{n\to\infty} (-1)^n n^3 \neq 0$

$x = \frac{7}{2}$ $\quad \sum \frac{n^3(7-5)^n}{2^n} = \sum n^3$

Diverges

$\lim_{n\to\infty} n^3 \neq 0$

11) For what values of x does the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$ converge?

$$\lim_{n\to\infty}\left|\frac{(x-1)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{(x-1)^n}\right| = \left|\frac{(x-1)\sqrt{n}}{\sqrt{n+1}}\right| = |x-1| < 1$$

$$-1 < x-1 < 1$$

$$\boxed{0 \leq x < 2} \quad \text{I.O.C}$$

$$\text{R.O.C} = 1$$

$x = 0$ $\quad \sum \frac{(-1)^n}{\sqrt{n}}$

$\lim_{n\to\infty}\left|\frac{(-1)^n}{\sqrt{n}}\right| = 0$

$\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$

$\boxed{\text{conditional}}$

Absolute? NO

$\sum \frac{1}{\sqrt{n}}$ $\quad p = \frac{1}{2} < 1$ diverges

$x = 2$

$\sum \frac{1}{\sqrt{n}}$ $\quad p = \frac{1}{2} < 1$ diverges

12) a) Find all values of x for which the geometric series $\sum_{n=0}^{\infty} e^{nx}$ converges.

b) Find the function (sum) represented by the series $\sum_{n=0}^{\infty} e^{nx}$

c) Find all values of x for which $\sum_{n=0}^{\infty} e^{nx} > 2$

d) Find all values of x for which $\sum_{n=0}^{\infty} e^{nx} < 1$

13) Let $f(x) = \sum_{n=0}^{\infty} \frac{(-2)^n x^n}{3^n (n+1)^2}$

(a) Find the interval of convergence of the series.

(b) For what values of x does the series converge absolutely?

(c) Find the domain (interval of convergence) of the following function:
$h(x) = f(x^2)$.