* If the series your comparing to converges original ≤ comparison * If the series your comparing to diverges original ≥ comparison	Direct Comparison Test original A) $\sum_{n=1}^{\infty} \frac{1}{n^2+9}$ converges $\frac{1}{n^2} \geq \frac{1}{n^2+9}$ $\frac{1}{n^2+9} \leq \frac{1}{n^2}$ compare to $\sum \frac{1}{n^2}$ $p=2>1$ converges comparison B) $\sum_{n=1}^{\infty} \frac{n}{n^2-9}$ original diverges Comparison $\sum \frac{1}{n}$ Harmonic diverges $(p=1 \leq 1)$ original ≥ comparison $\lim_{b\to\infty} \int_1^b \frac{x}{x^2+9}\,dx = \frac{1}{2}\ln(x^2+9)\Big]_1^b = \frac{1}{2}\ln(b^2+9) - \ln(10) =$ Diverge C) $\sum_{n=1}^{\infty} \frac{n}{n^2+9}$ original diverges by Direct Comparison only if original > comparison Compare to $\sum \frac{1}{n}$ Harmonic diverges $\frac{n}{n^2+9} \geq \frac{1}{n}$ False $\frac{1}{10} + \frac{2}{13} + \frac{3}{18} + \frac{4}{25} \geq 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

Limit Comparison Test

A) $\sum_{n=1}^{\infty} \frac{n}{n^2+9}$ diverges

Comparison $\sum \frac{1}{n}$ Harmonic Diverges

$$\lim_{n\to\infty} \frac{\left(\frac{n}{n^2+9}\right)}{\frac{1}{n}} = \frac{n^2}{n^2+9} = \boxed{1}$$

If the result of the LCT is a constant both series either diverge or converge

B) $\sum_{n=1}^{\infty} \frac{n}{n^2-9}$ diverges

Compare to $\sum \frac{1}{n}$ Harmonic diverges

$$\lim_{n\to\infty} \frac{\frac{n}{n^2-9}}{\frac{1}{n}} = \frac{n^2}{n^2-9} = 1$$

C) $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$ converges

Compare $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$p=2>1$ converges

$$\lim_{n\to\infty} \frac{\frac{1}{n^2+4}}{\frac{1}{n^2}} = \frac{n^2}{n^2+4} = 1$$

D) $\cancel{\sum_{n=1}^{\infty} \frac{1}{n^2+4}}$

E) $\sum_{n=1}^{\infty} \frac{5+3^n}{4+5^n}$ converges

Compare to $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$ $|r| = \frac{3}{5} < 1$ converges

$$\lim_{n\to\infty} \frac{\left(\frac{5+3^n}{4+5^n}\right)}{\left(\frac{3^n}{5^n}\right)} =$$

$$\lim_{n\to\infty} \frac{5^n(5+3^n)}{3^n(4+5^n)} = 1$$

$$\frac{5^n \cdot 3^n}{3^n \cdot 5^n} = \frac{15^n}{15^n}$$