Series

2016 BC 6

The function *f* has a Taylor series about x = 1 that converges to f(x) for all x in the interval of convergence. It is known that f(1) = 1, $f'(1) = \frac{-1}{2}$, and the nth derivative of *f* at x = 1 is given by $f^{n}(1) = (-1)^{n}\frac{(n-1)!}{2^{n}}$ for $n \geq 2$.

a) Write the first 4 non-zero terms and the general term of the Taylor series for f about x = 1

2015 BC6

1. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty}\frac{(-3)^{n-1}}{n}x^{n} = x - \frac{3}{2}x^{2} + 3x^{3} - \cdots + \frac{(-3)^{n-1}}{n}x^{n}\cdots$ and converges to f(x) for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

   a) Write the first four non-zero terms of the Maclaurin series for f', the derivative of f. Express f' as a rational function for $|x| < R$.

   b) Write the first four nonzero terms of the Maclaurin series for e^x. Use the Maclaurin series for e^x to write the third-degree polynomial for $g(x) = e^x f(x)$ about x = 0.

2014 BC6

6. The Taylor series for a function f about x = 1 is given by $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-1)^n$ and converges to f(x) for $|x-1|<R$, where r is the radius of convergence of the Taylor series.

b) Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about x = 1.

c) The Taylor series for f' about x = 1, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1|<R$. Use this function to determine f for $|x-1|<R$.

2009 BC#5

Consider the differential equation $\frac{dy}{dx}=6x^2-x^2y$. Let y = f(x) be a particular solution to this differential equation with the initial condition f(-1) = 2.

At the point (-1, 2), the value of $\frac{d^2y}{dx^2}$ is -12. Find the second-degree Taylor Polynomial for f about x = -1.

2009 BC#6

The Maclaurin series for e^x is $e^x=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\cdots+\cdots\frac{x^n}{n!}+\cdots$. The continuous function f is defined by $f(x)=\frac{e^{(x-1)^2}-1}{(x-1)^2}$ for $x\neq 1$ and $f(1)=1$. The function has derivatives of all orders x= 1.

a. Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about x = 1.

b. Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about x = 1.

c. Use the Taylor Series to determine whether the graph of f has any inflections points.

2012 #4

The function f is twice differentiable for x > 0 with f(1) = 15 and $f''(1) = 20$. Values f', the derivative of f, are given for selected values of x in the table.

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

a) Write the second-degree Taylor polynomial for f about x = 1. Use the Taylor polynomial to approximate f(1.4).

The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots$$

a) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$

2008 BC6 Form B

6c. The derivative of ln(1 + x^2) is $\frac{2x}{1+x^2}$. Write the first four nonzero terms of the Taylor series for ln(1 + x^2).

2006 FormB BC6

The function f is defined by $f(x) = \frac{1}{1+x^3}$.

a. Find the first four nonzero terms and general term for the Maclaurin series for f.

b. Find the first three nonzero terms and the general term for $f'(x)$.

2013 BC6

A function f has derivatives of all orders at x = 0. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about x = 0.

a) It is known that f(0) = -4 and that $P_1\left(\frac{1}{2}\right)$=-3. Show that $f'(0)=2$.

b) It is known that $f''(0)=\frac{-2}{3}$ and $f'''(0)=\frac{1}{3}$. Find $P_3(x)$

c) The function h has first derivative given by $h'(x)=f(2x)$. It is known that h(0) = 7. Find the third degree Taylor polynomial for h about x = 0.

2011 BC6

Let $f(x)=\sin\left(x^2\right)+\cos x$.

a. Write the first four nonzero terms of the Taylor series for sinx about x = 0, and write the first four nonzero terms of the Taylor series for sin(x^2) about x = 0.

b. Write the first four nonzero terms of the Taylor series for cosx about x = 0. Use this series and the series for sin(x^2), found in part a, to write the first four nonzero terms of the Taylor series for f(x) about x = 0.

c. Find the value of $f^{(6)}(0)$.

14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor Polynomial about x = 0 for sin x?

$(A)\ 1-\frac{1}{2}+\frac{1}{24}$ $(B)\ 1-\frac{1}{2}+\frac{1}{4}$ $(C)\ 1-\frac{1}{3}+\frac{1}{5}$ $(D)\ 1-\frac{1}{4}+\frac{1}{8}$ $(E)\ 1-\frac{1}{6}+\frac{1}{120}$

89. The graph of the function represented by the Maclaurin series $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots+\frac{(-1)^n x^n}{n!}+\cdots$ intersects the graph of y = x^3 at x =

(A) .773 (B) .865 (C) .929 (D) 1.000 (E) 1.857

17. Let f be the function given by f(x) = ln(3 – x). The third degree Taylor polynomial for f about x = 2 is

$(A)\ -(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$ $(B)\ -(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

$(C)\ (x-2)+(x-2)^2+(x-2)^3$ $(D)\ (x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

$(E)\ (x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

24. The Taylor series for sin x about x = 0 is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$ If f is a function such that $f'(x)=\sin(x^2)$, then the coefficient of x^7 in the Taylor series for f(x) about x = 0 is

$(A)\ \frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $\frac{-1}{42}$ (E) $\frac{-1}{7!}$

77. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

A) -30 B) -15 C) -5 D) $\frac{-5}{6}$ E) $-\frac{1}{6}$

28. What is the coefficient of x^2 in the Taylor Series $\frac{1}{(1+x)^2}$ about $x = 0$?

A) $\frac{1}{6}$ B. $\frac{1}{3}$ C) 1 D) 3 E) 6

20. A function f has Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}+\cdots$. Which of the following is an expression for f(x)?

A) $-3x\sin x + 3x^2$
B) $-\cos(x^2) + 1$
C) $-x^2\cos x + x^2$
D) $x^2e^x - x^3 - x^2$
E) $e^{x^2} - x^2 - 1$

11. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

A) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$
B) $x^2 + x^3 + x^4 + x^5 + \cdots$
C) $x^2 + 2x^3 + 3x^4 + 4x^5 + \cdots$
D) $x^2 + x^4 + x^6 + x^8 + \cdots$
E) $x^2 - x^4 + x^6 - x^8 + \cdots$

23. If f(x) = xsin(2x), which of the following is the Taylor Series for f about x = 0?

A) $x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots$

B) $x-\frac{4x^3}{2!}+\frac{16x^5}{4!}-\frac{64x^7}{6!}+\cdots$

C) $2x-\frac{8x^3}{3!}+\frac{32x^5}{5!}-\frac{128x^7}{7!}+\cdots$

D) $2x^2-\frac{2x^4}{3!}+\frac{2x^6}{5!}-\frac{2x^8}{7!}+\cdots$

E) $2x^2-\frac{8x^4}{3!}+\frac{32x^6}{5!}-\frac{128x^8}{7!}+\cdots$

84. Let f be a function with f(3) =2, $f'(3)=-1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the the third degree Taylor Polynomial for f about x =3?

A) $2 - (x - 3) + 3(x - 3)^2 + 2(x-3)^3$

B) $2 - (x - 3) + 3(x - 3)^2 + 4(x-3)^3$

C) $2 - (x - 3) + 6(x - 3)^2 + 12(x-3)^3$

D) $2 - x + 3x^2 + 2x^3$

E) $2 - x + 6x^2 + 12x^3$