

Error

83. The Taylor Series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $.3 \leq x \leq 1.7$ is

- (A) .030 (B) .039 (C) .145 (D) .153 (E) .529

2010 BC #6

The Taylor series $g(x) = 1 - \frac{x}{2!} + \frac{x^3}{3 \cdot 4!} - \frac{x^5}{5 \cdot 6!}$ about $x = 0$, evaluated at $x = 1$, is an alternating series with individual terms that decrease in absolute value to 0. Use the third degree polynomial to Estimate the value of $g(1)$. Explain why this estimate differs from the actual value of $g(1)$ by less than $\frac{1}{6!}$.

2012 #6

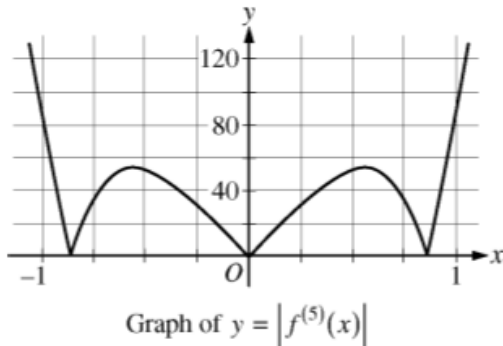
The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \dots$$

- a) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

2011 BC6

Let $f(x) = \sin(x^2) + \cos x$. The graph of $y = |f^{(5)}(x)|$ is shown.



Let $P_4(x)$ be the fourth degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$, shown above, show that $\left| P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right) \right| < \frac{1}{3000}$.

2008 BC 6 Form B

6. The first 4 nonzero terms of the Taylor Series for $\ln(1 + x^2)$ are

$$x^2 - \frac{1}{2}x^4 + \frac{1}{3}x^6 - \frac{1}{4}x^8$$

d. Use this series to find a rational number A such that $\left| A - \ln\left(\frac{5}{4}\right) \right| < \frac{1}{100}$. Justify your answer.

2007 BC6

6. The first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2} dt$ about $x = 0$ is

$$x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42}.$$

c. Find $\int_0^{1/2} e^{-t^2} dt$ using the first 2 terms.

d. Explain why the estimate found in part c, differs from the actual value of $\int_0^{1/2} e^{-t^2} dt$ by less than $\frac{1}{200}$.

2006 FormB BC6

The function f is defined by $f(x) = \frac{1}{1+x^3}$.

- c. Find the first four nonzero terms and the general term for the Maclaurin series representing $\int_0^x f(t)dt$.
- D. Use the first three nonzero terms of the infinite series found in part (C) to approximate $\int_0^{1/2} f(t)dt$. What are the properties of the terms of the series representing $\int_0^{1/2} f(t)dt$ that this approximation is within $\frac{1}{10000}$ of the exact value of the integral.