CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy Chapter 9: Error and Series

What you'll Learn About How to find the error of a series that does not alternate

Lagrange Error Bound/Taylors Inequality/Remainder Estimation Theorem

- Give the first term of the series for $f(x) = e^x$ centered at x = 01.
- 2. Find the approximation for P(.1) = 1
- Find f(.1) = e = 1.101570918 3.
- How accurate is the approximation. 4.
- 5. What is the value of the next term of the polynomial at x = .1

- Give the first two terms of the series for $f(x) = e^x$ centered at x = 0
- 2. Find the approximation for P(.1)
- 3. Find f(.1)
- (e.1- g(x)=.005176918 How accurate is the approximation.
- 5. What is the value of the next term of the polynomial at

- 1. Give the first three terms of the series for $f(x) = e^x$ centered at x = 0
- 2. Find the approximation for P(.1)
- 3. Find f(.1)
- How accurate is the approximation. $|e^{-1}-\rho_3(x)|=.000(70918)$ 4.
- What is the value of the next term of the polynomial at x = .1 (000 | 6 Give the first 4 terms of the series for f(x) e centered at x

1) 4 Terms ex centered at x=0

 $1 + x + x^2 + x^3$

.00000460487

 $|f(x)-P(x)| \le R$ Where R = (Max of thenext derivative on the given
interval (n+1)!

Where x-c is the distance from the center

Where n is the order

We must build the next term a little bit bigger to have a good boundary for the error.

Remember, whenever you see this, $|f(x)-P(x)| \le R$, you are finding error bound

whenever you see this, |f(x)-P(x)|, you are finding the actual error between the function and the approximation from the polynomial

2. Use Taylors Inequality to determine the error bound $|f(x)-P(x)| \le R$ from $f(x) = e^{-x}$ $f(x) = e^{-x}$

 $t_n(x) = e_x$ $t_n(x) = e_x$

F4(1)x4 Property (-1)4 Property (-1)

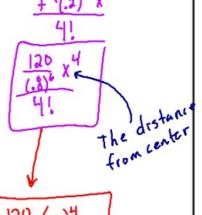
1. Find the 3rd order polynomial of the series for $f(x) = \frac{1}{(1-x)^2}$ centered at

 $f(x) = (1-x)^{-2}$ $f'(x) = +2(1-x)^{-3}$ f''(0) = 2x f''(0) = 6 $f'''(x) = +24(1-x)^{-5}$ f'''(0) = 24

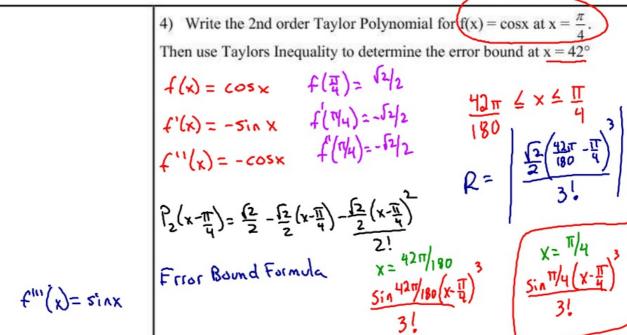
 $P_3(x) = 1 + 2x + \frac{6x^2}{2} + \frac{24x^3}{316}$

2. Find the Lagrange error bound $|f(x) - P(x)| \le R$ for the series between $0 \le x \le .2$ Build Formula

 $\frac{4i}{130x_{4}}$ $= \frac{(1-x)_{6}}{150}$ $= \frac{4i}{150}$ $\frac{130x_{4}}{11}$ $= \frac{150}{150}$ $= \frac{150}{150}$ $= \frac{150}{150}$

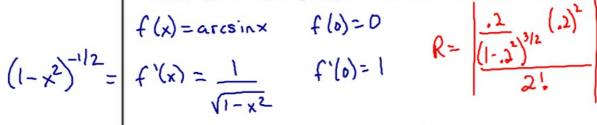


23 | Page



f"(x)= sinx

5) Write the 1st degree Taylor Polynomial for $f(x) = \arcsin x$ at x = 0. Then use Taylors Inequality to determine the error bound at x = .2



 $(1-x^{2})^{-1/2} = \begin{cases} f(x) = \arcsin x & f(x) = 0 \\ f'(x) = \frac{1}{\sqrt{1-x^{2}}} & f'(x) = 1 \end{cases}$ $f''(x) = -\frac{1}{2}(1-x^{2})^{3/2}$ $f''(x) = \frac{1}{\sqrt{1-x^{2}}} \qquad \begin{cases} f'(x) = 1 \\ (1-x^{2})^{3/2} \end{cases}$ $f''(x) = -\frac{1}{2}(1-x^{2})^{3/2}$ $f'''(x) = \frac{x}{(1-x^{2})^{3/2}}$ $f'''(x) = \frac{x}{(1-x^{2})^{3/2}}$ $f'''(x) = \frac{x}{2!}$ $f'''(x) = \frac{x}{2!}$

Summary of Error Bound

For an Alternating Series - Use the next term

For a series that is Not Alternating

- 1. Write down the formula for the next derivative.
- 2. Find the value of the next derivative at the ends of the interval and the center.
- 3. Whichever value is bigger is the value you use to build your error bound term

$f(x) = \sum_{n=0}^{\infty} c_n x^n$	INTERVAL OF CONVERGENCE	RADIUS OF CONVERGENCE
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	(-1, 1)	1
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$(-\infty,\infty)$	∞
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	$(-\infty,\infty)$	∞
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	$(-\infty,\infty)$	∞
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$ $= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	(-1,1]	1
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ $= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	(-1,1	1