

Chapter 7

Review Video

$$\text{p. 386 #4} \quad v(t) = 6t^2 - 18t + 12 \quad 0 \leq t \leq 2$$

- a. Determine when the particle is moving to the right, to the left, and stopped

$$v(t) = 0$$

$$0 = 6t^2 - 18t + 12$$

$$0 = t^2 - 3t + 2$$

$$0 = (t-2)(t-1)$$

$$t=2 \quad t=1$$

$$v(0) = 12 > 0 \quad \text{right} \quad (0, 1)$$

$$v\left(\frac{3}{2}\right) = 6\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 12$$

$$= 6\left(\frac{9}{4}\right) - 27 + 12 \quad \text{Left}$$

$$= \frac{27}{2} - 27 + 12 \quad (1, 2)$$

$$= 13.5 - 27 + 12 < 0$$

p. 386 #4 $v(t) = 6t^2 - 18t + 12$ $0 \leq t \leq 2$

b. Find the particle's displacement for the given time interval.

$$\begin{aligned} \int_0^2 6t^2 - 18t + 12 &= \left[2t^3 - 9t^2 + 12t \right]_0^2 \\ &= \left[2(2)^3 - 9(2)^2 + 12(2) \right] - [0] \\ &= 16 - 36 + 24 \\ &= 4 \text{ meters} \end{aligned}$$

p. 386 #4 $v(t) = 6t^2 - 18t + 12$ $0 \leq t \leq 2$

c. If $s(0) = 3$, what is the particle's final position?

3 + 4 = 7 meters

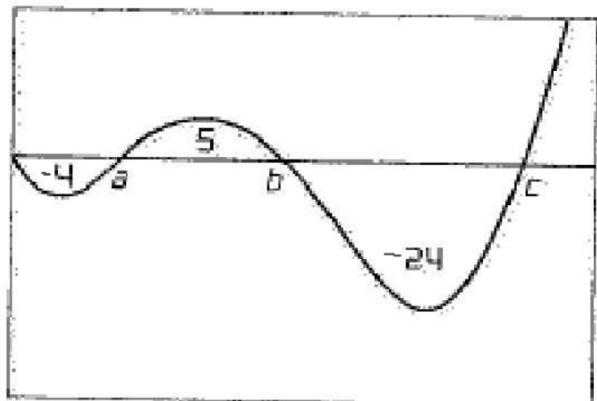
$$p. 386 \#4 \quad v(t) = 6t^2 - 18t + 12 \quad 0 \leq t \leq 2$$

d. Find the total distance traveled by the particle.

$$\begin{aligned} \int_0^2 |v(t)| dt &= \int_0^1 v(t) dt + \int_1^2 |v(t)| dt \\ &= \left[2t^3 - 9t^2 + 12t \right]_0^1 + \left| 2t^3 - 9t^2 + 12t \right|_1^2 \\ &= (5) + |(4) - (5)| \\ &= 6 \text{ meters} \end{aligned}$$

p. 386 #12-16

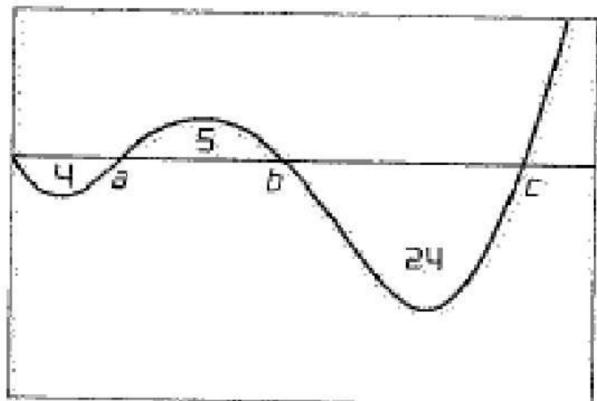
12. What is the particle's displacement between $t = 0$ and $t = c$



$$-4 + 5 - 24 = -23$$

p. 386 #12-16

13. What is the total distance traveled between $t = 0$ and $t = c$

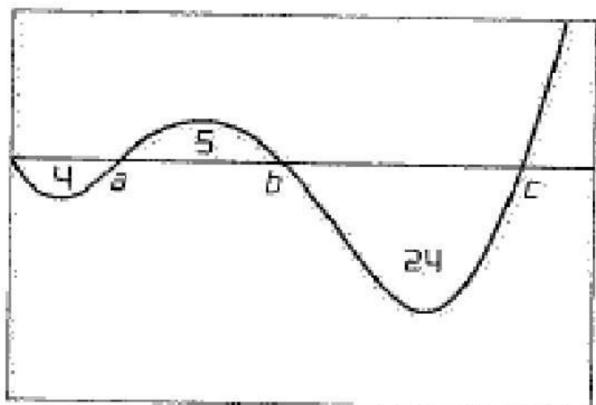


$$4 + 5 + 24 = 33$$

p. 386 #12-16

$$s(0) = 15$$

14. Give the positions of the particle at times a, b, and c.



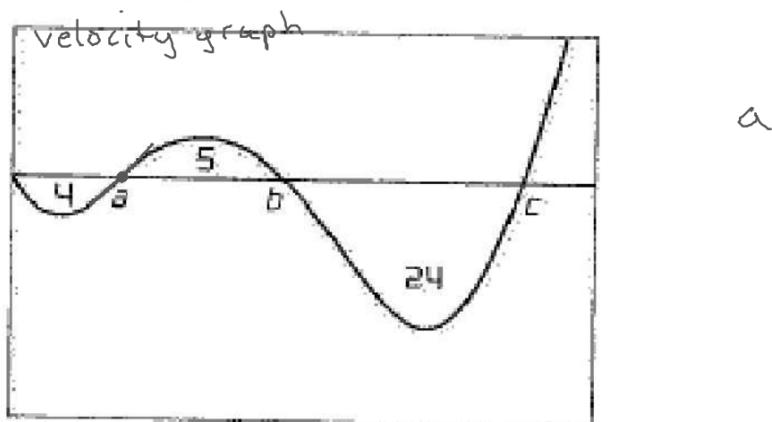
$$s(a) = 15 - 4 = 11$$

$$s(b) = 11 + 5 = 16$$

$$s(c) = 16 - 24 = -8$$

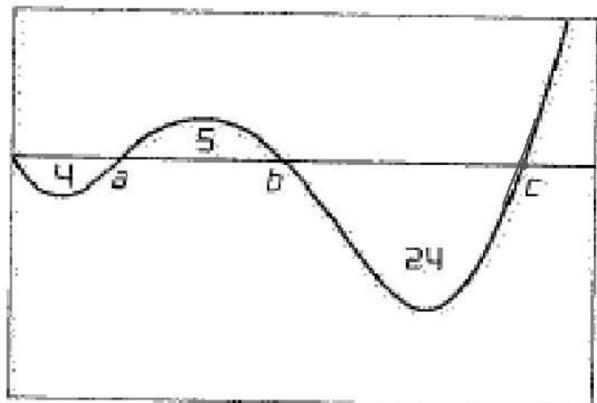
p. 386 #12-16

15. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$



p. 386 #12-16

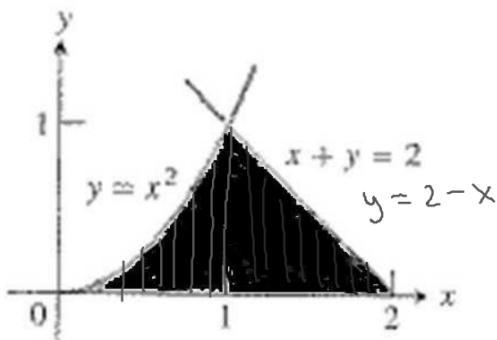
15. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$



c

p. 396 #10 Find the area of the shaded region

10.



$$A = \int_0^1 x^2 - 0 + \int_1^2 (2 - x) - 0$$

$$A = \left[\frac{1}{3}x^3 \right]_0^1 + \left[2x - \frac{1}{2}x^2 \right]_1^2$$

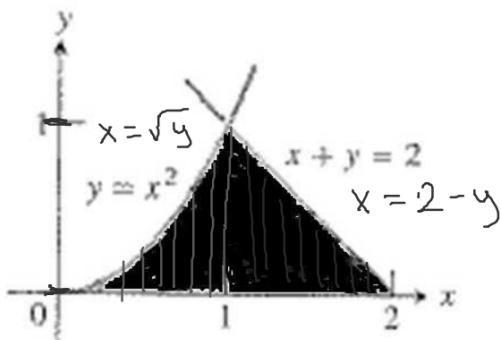
$$= \frac{1}{3} + \left((4 - 2) - \left(2 - \frac{1}{2} \right) \right)$$

$$= \frac{1}{3} + (2 - 1.5)$$

$$\frac{1}{3} + \frac{1}{5} = \frac{2}{7} + \frac{3}{7} \approx 5$$

p. 396 #10 Find the area of the shaded region

10.



$$\begin{aligned}
 A &= \int_0^1 (2-y) - y^{1/2} \\
 &= \left[2y - \frac{1}{2}y^2 - \frac{2}{3}y^{3/2} \right]_0^1 \\
 &= 2 - \frac{1}{2} - \frac{2}{3} \\
 &= \frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \boxed{\frac{5}{6}}
 \end{aligned}$$

$$7 - 2x^2 = x^2 + 4$$

p. 396 17

$$3 = 3x^2$$

$$1 = x^2$$

$$x = \pm 1$$

Find the area of the region enclosed by the lines and curves

opens down

$$y = 7 - 2x^2$$

top

opens up

$$y = x^2 + 4$$

bottom

$$A = \int_{-1}^1 -3x^2 + 3$$

$$A = \left[-x^3 + 3x \right]_{-1}^1$$

$$A = (-1+3) - (1-3)$$

$$2 - (-2) = 4$$

$$A = \int (7 - 2x^2) - (x^2 + 4)$$

$$= \int 7 - 2x^2 - x^2 - 4$$

$$y^2 = 3 - 2y^2$$

p. 396 24 $3y^2 = 3$ $y^2 = 1$ $y = \pm 1$

Find the area of the region enclosed by the lines and curves

$$x - y^2 = 0 \quad \text{and} \quad x + 2y^2 = 3$$

$$\begin{array}{l} x = y^2 \\ \cancel{\text{Right}}? \text{ Let } + \\ \cancel{x = 3 - 2y^2} \\ \text{Left}+? \\ \text{Right} \end{array}$$

$$\int y^2 - (3 - 2y^2)$$

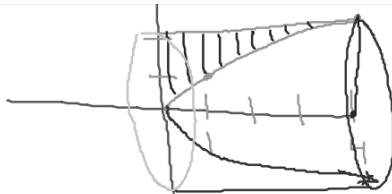
$$-\int y^2 - 3 + 2y^2$$

$$A = - \int_{-1}^1 3y^2 - 3$$

$$A = - \left[y^3 - 3y \right]_{-1}^1$$

$$A = - \left[(1 - 3) - (-1 + 3) \right] \\ - \left[-2 - 2 \right] = +4$$

p. 407 29a

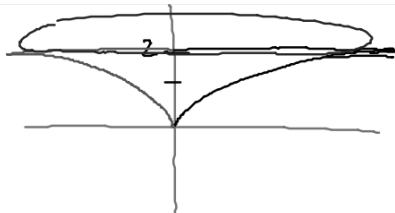


Find the volume of the solid generated by revolving the region
bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the x-axis

$$V = \pi \int_0^4 (2)^2 - \pi \int_0^4 (\sqrt{x})^2$$

$$\begin{aligned} V &= \pi \int_0^4 4 - \pi \int_0^4 x \\ &= \pi [4x]_0^4 - \pi \left[\frac{1}{2}x^2 \right]_0^4 = 16\pi - 8\pi \end{aligned}$$

p. 407 29b



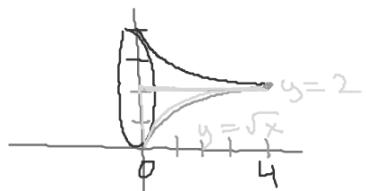
Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the y-axis

$$y = \sqrt{x}$$

$$y^2 = x$$

$$V = \pi \int_0^2 (y^2)^2 dy = \pi \int_0^2 y^4 dy = \pi \left[\frac{1}{5} y^5 \right]_0^2 = \pi \left[\frac{32}{5} \right] = \frac{32\pi}{5}$$

p. 407 29c



$$\begin{aligned} 4^{3/2} &= \sqrt{4^3} \\ &= \sqrt{64} \\ &= 8 \end{aligned}$$

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the line $y = 2$

$$V = \pi \int_0^4 (2 - \sqrt{x})^2$$

$$V = \pi \int_0^4 (2 - \sqrt{x})(2 - \sqrt{x})$$

$$V = \pi \int_0^4 (4 - 4\sqrt{x} + x)$$

$$V = \pi \int_0^4 (4 - 4x^{1/2} + x)$$

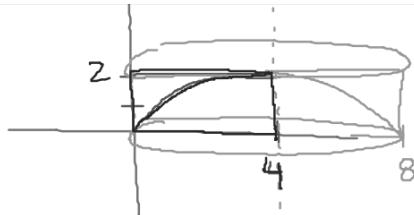
$$V = \pi \left[4x - 4 \cdot \frac{2}{3} x^{3/2} + \frac{1}{2} x^2 \right]_0^4$$

$$V = \pi \left[16 - \frac{64}{3} + 8 \right]$$

$$= \pi [24 - \underline{64}] = \underline{72} - \underline{64} \cancel{\pi}$$

p. 407 29d

$$x = y^2$$



Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the line $x = 4$

cylinder - bowl

$$V = \pi \int_0^2 4^2 - \pi \int_0^2 (4-y^2)^2$$

$$V = 32\pi - \pi \left[32 - \frac{64}{3} + \frac{32}{5} \right]$$

$$V = 32\pi - \frac{256\pi}{15}$$

$$V = \pi \int_0^2 16 - \pi \int_0^2 16 - 8y^2 + y^4$$

$$V = \pi [16y]_0^2 - \pi [16y - \frac{8}{3}y^3 + \frac{1}{5}y^5]_0^2$$

$$V = \frac{224\pi}{15}$$