

Total Distance/Total Amount/Position(Cartesian) – Free Response Pieces

2010 #2

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t=0$) and 8 P.M. ($t=8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \leq t \leq 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table.

$t(\text{hours})$	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function, P , where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \leq t \leq 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?

2009 #2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \leq t \leq 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

How many people are in the auditorium when the concert begins?

2011 #2

At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t , is modeled by a differentiable function B for which it is known that $B'(t) = 13.84e^{-1.73t}$. Using the given model, at time $t = 10$, what is the temperature of the biscuits..

2013 BC1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and

$0 \leq t \leq 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate of 100 tons per hour.

b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

2012 #1

t(minutes)	0	4	9	15	20
W(t) degrees F	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

c) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = .4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

2014 BC4

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/min)	0	100	40	-120	-150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$.

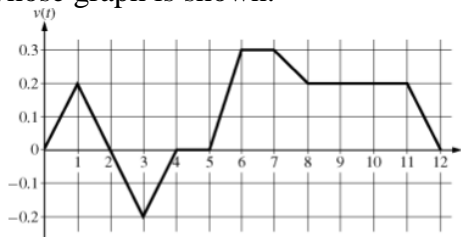
Johanna jogs along a straight path. For $0 \leq t \leq 40$. Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

- a) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.

2009 #1

Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \leq t \leq 12$ minutes, her velocity, $v(t)$, in miles per minute is modeled by the piecewise-linear function whose graph is shown.



Using correct units, explain the meaning of $\int_0^{12} |v(t)| dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)| dt$.

Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{15} t\right)$, where $w(t)$ is in miles per minute for $0 \leq t \leq 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

2016 BC 1

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table below. At time $t = 0$, there are 50,000 liters of water in the tank.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

- a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- b) Use a left Riemann sum with four subintervals indicated by the table to estimate the total amount of water removed during the 8-hour time interval $0 \leq t \leq 8$. Is this an overestimate or underestimate of the total amount of water removed. Give a reason for your answer.
- c) Use your answer in part (b) to find an estimate of the total amount of water in the tank, to the nearest liter at the end of 8 hours.
- d) For $0 \leq t \leq 8$ is there a time, t , when the rate at which water is pumped into the tank is the same as the rate at which the water is removed from the tank. Explain why or why not?

2015 BC1

1. The rate at which rainwater flows into a drainpipe is modeled by the function R , where

$$R(t) = 20\sin\left(\frac{t^2}{35}\right) \text{ cubic feet per hour, } t \text{ is measured in hours, and } 0 \leq t \leq 8. \text{ The pipe is}$$

partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -.04t^3 + .4t^2 + .96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$.
- b) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- c) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.