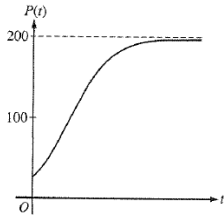Logistic Growth Functions

2008 BC24

24.   Which of the following differential equations for a population P could model the logistic growth
        shown in the figure above?

A) $\dfrac{dP}{dt} = 0.2P - 0.001P^2$ B) $\dfrac{dP}{dt} = .1P - .0001P^2$

C) $\dfrac{dP}{dt} = 0.2P^2 - 0.001$ D) $\dfrac{dP}{dt} = 0.1P^2 - .001P$

E) $\dfrac{dP}{dt} = 0.1P^2 - 0.001$

2003 BC21

21.   The number of moose in a national park is modeled by the function M that satisfies the logistic
        differential equation $\dfrac{dM}{dt} = .6M\left(1 - \dfrac{M}{200}\right)$, where t is the time in years and M(0)=50. What is
        the $\lim\limits_{t\to\infty} M(t)$?

        A) 50   B) 200   C) 500   D) 1000   E) 2000

84. The rate of change, $\dfrac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1200. At 10 A.M., the number of people on the beach is 200 and is increasing at the rate of 400 people per hour. Which of the following differential equations describes the situation.

A) $\dfrac{dP}{dt} = \dfrac{1}{400}(1200 - P) + 200$ B) $\dfrac{dP}{dt} = \dfrac{2}{5}(1200 - P)$

C) $\dfrac{dP}{dt} = \dfrac{1}{500}P(1200 - P)$ D) $\dfrac{dP}{dt} = \dfrac{1}{400}P(1200 - P)$

E) $\dfrac{dP}{dt} = 400P(1200 - P)$

26. The population P(t) of a species satisfies the logistic differential equation $\dfrac{dP}{dt} = P\left(2 - \dfrac{P}{5000}\right)$, where the initial position P(0)=3000 and t is the time in years. What is $\lim\limits_{t \to \infty} P(t)$?

A) 2500   B) 3000   C) 4200   D) 5000   E) 10000

2006B BC5

Let g be a function with g(4) = 1, such that all points (x, y) on the graph of g satisfy the logistic differential equation $\dfrac{dy}{dx} = 2y(3 - y)$.

b) Given that g(4) = 1, find $\lim\limits_{x \to \infty} g(x)$ and $\lim\limits_{x \to \infty} g'(x)$.

(It is not necessary to solve for g(x) or to show how you arrived at your answers.)

c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for g(x).)

2004 BC5

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

a) If P(0) = 3, what is the $\lim_{t \to \infty} P(t)$?

If P(0) = 20, what is the $\lim_{t \to \infty} P(t)$?

b) If P(0) = 3, for what value of P is the population growing the fastest?

2008 BC 6

Consider the logistic growth equation

$$\frac{dy}{dt} = \frac{y}{8}(6 - y)$$

Let y = f(t) be the particular solution to the differential equation y(0) = 8.

a) If y(0) = 8, what is the $\lim_{t \to \infty} y(t)$?

b) If y(0) = 1, for what value of y is the population growing the fastest?

c) A slope field for this differential equation is given below. Sketch possible solution curves through the points (3, 2) and (0, 8)
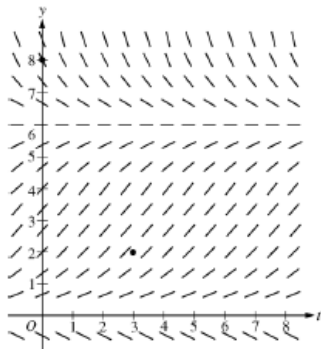
d) Use Euler's Method, starting at y(0) = 8, with 2 steps of equal size to approximate f(1)

Practice Test 2011 BC

14. Let k be a positive constant. Which of the following is a logistic differential equation?

A) $\dfrac{dy}{dt} = kt$

B) $\dfrac{dy}{dt} = ky$

C) $\dfrac{dy}{dt} = kt(1-t)$

D) $\dfrac{dy}{dt} = ky(1-t)$

E) $\dfrac{dy}{dt} = ky(1-y)$

Rogawski

8.   If y(x) is a solution to $\dfrac{dy}{dx} = 2y(5-y)$ with y(0) = 3 then as $x \to \infty$,

A)   y(x) increases to ∞

B)   y(x) increases to 5

C)   y(x) decreases to 5

D)   y(x) decreases to 2

E)   y(x) decreases to 0

9.   If y(x) is a solution to $\dfrac{dy}{dx} = 3y(6-y)$ with y(0) = 10 then as $x \to \infty$,

A)   y(x) increases to ∞

B)   y(x) increases to 6

C)   y(x) decreases to 6

D)   y(x) decreases to 3

E)   y(x) decreases to 0

16. If $\dfrac{dy}{dt} = 2y(12-3y)$ with y(0) = 1 then, y is increasing the fastest when

A)   y = 0

B)   y = 1

C)   y = 2

D)   y = 3

E)   y = 4

18. If $\dfrac{dy}{dt} = 2y(12-3y)$ with y(0) = 1, then the maximum value of y is

A)   y = 1

B)   y = 2

C)   y = 4

D)   y = 12

E)   Never attained; has no maximum value

Princeton Review (p. 806)

25.   Given the differential equation $\dfrac{dz}{dt} = z\left(4 - \dfrac{z}{100}\right)$, where z(0) = 50, what is the $\lim\limits_{t \to \infty} z(t)$?

A)   50   B)   100   C)   4   D) 400   E)   200

25.   Given the differential equation $\dfrac{dz}{dt} = z\left(4 - \dfrac{z}{100}\right)$, where z(0) = 50, then z is increasing the

fastest when z =

A)   50   B)   100   C)   4   D) 400   E)   200

Other Rate type problems

Rogawski

11.	The rate at which a certain disease spreads is proportional to the product of the percentage of the population with the disease and the percentage of the population that does not have the disease. If the constant of proportionality is .03, and y is the percent of people with the disease, then which of the following equations gives R(t), the rate at which the disease is spreading.

A) $R(t) = .03y$

B) $R(t) = .03\dfrac{dy}{dt}$

C) $R(t) = .03y(1 - y)$

D) $\dfrac{dr}{dt} = .03R(1 - R)$

E) $\dfrac{dr}{dt} = .03R$

2003 BC 12

12.	The rate of change of the volume, V, of water in a tank with respect to time, t is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship.

A) $V(t) = k\sqrt{t}$ 	**B)** $V(t) = k\sqrt{V}$ 	C) $\dfrac{dV}{dt} = k\sqrt{t}$

D) $\dfrac{dV}{dt} = \dfrac{k}{\sqrt{V}}$ 	E) $\dfrac{dV}{dt} = k\sqrt{V}$

12.	The rate of change of the volume, V, of water in a tank with respect to time, t is directly proportional to the inverse square root of the volume. Which of the following is a differential equation that describes this relationship.

A) $V(t) = k\sqrt{t}$ 	**B)** $V(t) = k\sqrt{V}$ 	C) $\dfrac{dV}{dt} = k\sqrt{t}$

D) $\dfrac{dV}{dt} = \dfrac{k}{\sqrt{V}}$ 	E) $\dfrac{dV}{dt} = k\sqrt{V}$

16.   Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population P(t) is increasing at rate directly proportional to $800 - P(t)$, where the constant of proportionality is k. Write the differential equation that describes this relationship.

23. If P(t) is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population.

A) $\dfrac{dP}{dt} = 200$ B) $\dfrac{dP}{dt} = 200t$ C) $\dfrac{dP}{dt} = 100t^2$

D) $\dfrac{dP}{dt} = 200P$ E) $\dfrac{dP}{dt} = 100P^2$

23. If P(t) is the size of a population at time t, which of the following differential equations describes quadratic growth in the size of the population.

A) $\dfrac{dP}{dt} = 200$ B) $\dfrac{dP}{dt} = 200t$ C) $\dfrac{dP}{dt} = 100t^2$

D) $\dfrac{dP}{dt} = 200P$ E) $\dfrac{dP}{dt} = 100P^2$

23. If P(t) is the size of a population at time t, which of the following differential equations describes cubic growth in the size of the population.

A) $\dfrac{dP}{dt} = 200$ B) $\dfrac{dP}{dt} = 200t$ C) $\dfrac{dP}{dt} = 100t^2$

D) $\dfrac{dP}{dt} = 200P$ E) $\dfrac{dP}{dt} = 100P^2$

23. If P(t) is the size of a population at time t, which of the following differential equations describes inverse variational growth in the size of the population.

A) $\dfrac{dP}{dt} = 200$ B) $\dfrac{dP}{dt} = 200t$ C) $\dfrac{dP}{dt} = 100t^2$

D) $\dfrac{dP}{dt} = 200P$ E) $\dfrac{dP}{dt} = 100P^2$