

# Chapter 6 Part 2

Review

$$1. \int_0^1 -15x^4(-3x^5 - 1)^5 dx \quad \text{Let } u = -3x^5 - 1$$

$$x=0 \quad u = -1$$

$$x=1 \quad u = -3-1 = -4$$

$$\int_{-1}^{-4} (-15x^4)(u^5) dx$$

$$\frac{du}{dx} = -15x^4$$

$$\int_{-1}^{-4} \left( \frac{du}{dx} \right) u^5 dx$$

$$\boxed{\int_{-1}^{-4} u^5 du}$$

$$3. \int_0^{\pi/4} \underbrace{4 \sec(4x) \tan(4x)}_{\text{circled}} \sec^4(4x) dx$$

$$\text{Let } u = \sec(4x)$$

$$x=0 \quad u = \sec(0) = 1$$

$$x=\frac{\pi}{4} \quad u = \sec\pi = -1$$

$$\frac{du}{dx} = 4 \sec(4x) \tan(4x)$$

$$\int_1^{-1} \left(\frac{du}{dx}\right) (u^4) dx$$

$$\boxed{\int_1^{-1} u^4 du}$$

Evaluate the integral

p. 338 33

$$\int \frac{\ln^6 x}{x} dx = \int \underline{(\ln x)^6} \cdot \frac{1}{x} dx = \boxed{\frac{1}{7} (\ln x)^7 + C}$$

$\uparrow$

$\checkmark \quad 7(\ln x)^6 \cdot \frac{1}{x}$

Evaluate the integral

p. 338 41

$$\int \frac{x}{x^2 + 1} dx = \boxed{\frac{1}{2} \ln(x^2 + 1) + C}$$

✓  $\frac{1}{x^2+1} \cdot 2x$

Evaluate the integral

p. 338 34

$$\int \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx = \boxed{\frac{1}{4} \tan^8\left(\frac{x}{2}\right) + C}$$

✓  $\frac{8 \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) \cdot \frac{1}{2}}{\boxed{4 \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right)}}$

Evaluate the integral

p. 338 37

$$\int \frac{\sin(2x+1)}{\cos^2(2x+1)} dx = \int \sin(2x+1) [\cos(2x+1)]^{-2}$$
$$= -\frac{1}{2} [\cos(2x+1)]^{-1} + C$$

$$\checkmark -1[\cos(2x+1)]^{-2} \cdot \sin(2x+1) \cdot 2$$

Evaluate the integral

p. 338 22

$$\int \frac{9x^2}{\sqrt{1-x^3}} dx = \int 9x^2 (1-x^3)^{-1/2} = \boxed{-6(1-x^3)^{1/2} + C}$$

$$\checkmark \frac{1}{2}(1-x^3)^{-1/2} \cdot (-3x^2)$$
$$\frac{-3x^2}{2\sqrt{1-x^3}} \quad \left(\frac{-2}{3}\right)^{-\frac{3}{2}} = 9\left(\frac{-2}{3}\right)$$

Evaluate the integral

p. 338 66

$$\int_0^2 \frac{e^x}{3+e^x} dx = \left. \ln|3+e^x| \right|_0^2 = \ln(3+e^2) - \ln(3+e^0)$$
$$= \boxed{\ln(3+e^2) - \ln 4}$$

$\frac{1}{3+e^x} \cdot e^x$

Find the average value on the given interval p. 338 53

$$\int_0^3 \sqrt{x+1} dx \quad \frac{1}{3} \int_0^3 (x+1)^{1/2} = \frac{1}{3} \left[ \frac{2}{3} (x+1)^{3/2} \right]_0^3$$
$$f(x) = \sqrt{x+1}$$
$$= \frac{1}{3} \left[ \frac{2}{3} (4)^{3/2} - \frac{2}{3} (1)^{3/2} \right]$$
$$4^{3/2} = \sqrt[2]{4^3} = \sqrt{64} = 8$$
$$\frac{1}{3} \left[ \frac{2}{3} (8) - \frac{2}{3} \right]$$
$$\frac{1}{3} \left[ \frac{14}{3} \right] = \left( \frac{14}{9} \right)$$

Evaluate the integral

p. 346 9

$$\int x \ln x dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{x} \cdot \frac{x^2}{2}$$

$$\frac{\ln x}{x} \left| \begin{array}{l} x dx \\ \frac{1}{x} \end{array} \right. = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

Evaluate the integral

p. 346 22

$$\int (x^2 - 5x)e^x dx = \boxed{(x^2 - 5x)(e^x) - (2x - 5)(e^x) + 2e^x + C}$$

$$\begin{array}{r} x^2 - 5x \\ 2x - 5 \\ \hline 2 \\ 0 \end{array} \quad \begin{array}{l} e^x \\ e^x \\ e^x \\ e^x \end{array}$$

Find the particular solution p. 357 5

$$\frac{dy}{dx} = (y-5)(x+2) \quad f(0)=1$$

$$\int \frac{dy}{y-5} = \int (x+2) dx$$

$$\ln|y-5| = \frac{1}{2}x^2 + 2x + C$$

$$\ln|-4| = C$$

$$\ln 4 = C$$

$$\ln(-y+5) = \frac{1}{2}x^2 + 2x + \ln 4$$

$$-y+5 = 4e^{\frac{1}{2}x^2 + 2x}$$

$$-y = 4e^{\frac{1}{2}x^2 + 2x} - 5$$

$$y = -4e^{\frac{1}{2}x^2 + 2x} + 5$$

Use Euler's method with 2 steps of equal value to estimate  $f(1)$ .

$$f(1) = -13$$

$$\frac{dy}{dx} = (y-5)(x+2) \quad f(0) = 1$$

$$(0, 1) \quad m = (-4)(2) = -8 \quad y = 1 - 8(0.5) = 1 - 4 = -3$$

$$(0.5, -3) \quad m = (-3-5)(0.5+2) = (-8)(2.5) = -20$$

$$(1, -13)$$

$$y = -3 - 20(0.5) = -3 - 10 = -13$$

Evaluate the integral

p. 369 6

$$\int \frac{2x+16}{x^2+x-6} dx$$

$$x=2 \quad 20 = 5B \quad B=4$$

$$x=-3 \quad 10 = -5A \quad A=-2$$

$$\int \frac{2x+16}{(x+3)(x-2)} = \int \left( \frac{-2}{(x+3)} + \frac{4}{(x-2)} \right) = -2 \ln|x+3| + 4 \ln|x-2| \\ = \ln(x+3)^{-2} + \ln(x-2)^4$$

$$\frac{2x+16}{(x+3)(x-2)} = \frac{A}{(x+3)} + \frac{B}{(x-2)} \rightarrow 2x+16 = A(x-2) + B(x+3)$$
$$\ln \left[ (x+3)^{-2} (x-2)^4 \right] + C = \ln \frac{4}{(x+3)^2} + C$$

Evaluate the integral

p. 369 16

$$\int \frac{2}{x^2 - 1} dx = \int \frac{-1}{(x+1)} + \frac{1}{(x-1)} = -\ln|x+1| + \ln|x-1|$$
$$\frac{2}{x^2-1} = \frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$
$$= \boxed{\ln \left| \frac{x-1}{x+1} \right| + C}$$

$$2 = A(x-1) + B(x+1)$$

$$x=1 \quad 2 = 2B \quad B=1$$

$$x=-1 \quad 2 = -2A \quad A=-1$$

Evaluate the integral

p. 467 24

$$\int_{-\infty}^{\infty} e^{2x} dx = \int_{-\infty}^0 e^{2x} + \int_0^{\infty} e^{2x} \rightarrow \text{Diverges}$$

$$\lim_{b \rightarrow -\infty} \int_b^0 e^{2x} + \lim_{b \rightarrow \infty} \int_0^b e^{2x}$$

$$\lim_{b \rightarrow -\infty} \left[ \frac{1}{2} e^{2x} \right]_b^0 + \lim_{b \rightarrow \infty} \left[ \frac{1}{2} e^{2x} \right]_0^b$$
$$\lim_{b \rightarrow -\infty} \left[ \frac{1}{2} - \frac{1}{2} e^{2b} \right] + \lim_{b \rightarrow \infty} \left[ \frac{1}{2} e^{2b} + \frac{1}{2} \right]$$

Evaluate the integral

p. 471 37

$$\int_1^{\infty} \frac{dx}{x^{3/2}}$$
$$\lim_{b \rightarrow \infty} \int_1^b x^{-3/2} = \lim_{b \rightarrow \infty} \left[ -2x^{-1/2} \right]_1^b = \lim_{b \rightarrow \infty} \left[ \frac{-2}{\sqrt{x}} \right]_1^b$$
$$= \lim_{b \rightarrow \infty} \left[ \frac{-2}{\sqrt{b}} - \left( \frac{-2}{\sqrt{1}} \right) \right]$$
$$0 + 2 = \boxed{2}$$

Evaluate the integral

p. 471 44

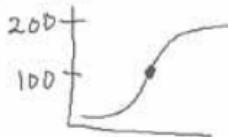
$$\int_3^{\infty} \frac{2dx}{x^2 - 2x} = \int -\frac{1}{x} + \frac{1}{x-2} = -\ln|x| + \ln|x-2|$$
$$= \ln \left| \frac{x-2}{x} \right|$$
$$\frac{2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$
$$2 = A(x-2) + Bx$$
$$x=2 \quad 2=2B \quad B=1$$
$$x=0 \quad 2=-2A \quad A=-1$$
$$\lim_{b \rightarrow \infty} \left[ \ln \left| \frac{x-2}{x} \right| \right]_3^b = \ln \left| \frac{b-2}{b} \right| - \ln \frac{1}{3}$$
$$= 0 - \ln \frac{1}{3}$$
$$= \boxed{-\ln \frac{1}{3}}$$

$$\frac{dM}{dt} = .6M\left(\frac{1}{200}\right)(200 - M)$$

## Logistic Growth

The number of moose in a national park is modeled by the function  $M$  that satisfies the logistic differential equation  $\frac{dM}{dt} = .6M\left(1 - \frac{M}{200}\right)$

where  $t$  is the time in years and  $M(0)=50$ . What is the  $\lim_{t \rightarrow \infty} M(t)$ ?



- A) 50      B) 200      C) 500      D) 1000      E) 2000

## Logistic Growth

The number of moose in a national park is modeled by the function  $M$  that satisfies the logistic differential equation  $\frac{dM}{dt} = .6M\left(1 - \frac{M}{200}\right)$

where  $t$  is the time in years and  $M(0)=50$ . For what value of  $M$  is the population growing the fastest?

- A) 50
- B) 100
- C) 200
- D) 500
- E) 1000