Chapter 6 Part 1

Test Review

At the beginning of 2010, a landfill contained 2200 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dw}{dt} = \frac{1}{20}(W - 200)$ for the next 20 years. W is measured in tons, t is measured in years from the start of 2010.

Use the line tangent to the graph of W at t = 0 to approximate the amount of solid waste that the landfill contains at the end of the first 6 months of 2010 $\left(t = \frac{1}{2}\right)$

t w

(0, 2200)

$$\frac{dw}{dt} = \frac{1}{20}(w - 200)$$

$$\frac{dw}{dt} = \frac{1}{20}(2200 - 200) = \frac{1}{20}(2000) = 100$$

$W = 2200 + 100(t - 0)$

$W = 2200 + 100\left(\frac{1}{2}\right)$

$W = 2200 + 50$

$W = 2250$ tons of waste

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$$\frac{dw}{dt} = \frac{1}{20}w - 10$$

$$\frac{d^2w}{dt^2} = \frac{1}{20}\frac{dw}{dt}$$

$$= \frac{1}{20}\left(\frac{1}{20}(w-200)\right)$$

$$\frac{d^2w}{dt^2} = \frac{1}{20}\left(\frac{1}{20}(2200-200)\right) > 0$$

W is concave up making the tangent line an underestimate

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Find the particular solution W=W(t) to the differential equation

$$\frac{dw}{dt} = \frac{1}{20}(W - 200)$$ with initial condition W(0) = $\overset{w}{2200}$.

$$dw = \left[\frac{1}{20}(w-200)\right]dt$$

$$\int \frac{dw}{w-200} = \int \frac{1}{20}\,dt$$

$$\ln(w-200) = \frac{1}{20}t + C$$

$$\ln(2000) = C$$

$$\ln(w-200) = \frac{1}{20}t + \ln 2000$$

$$w - 200 = e^{\frac{1}{20}t + \ln 2000}$$

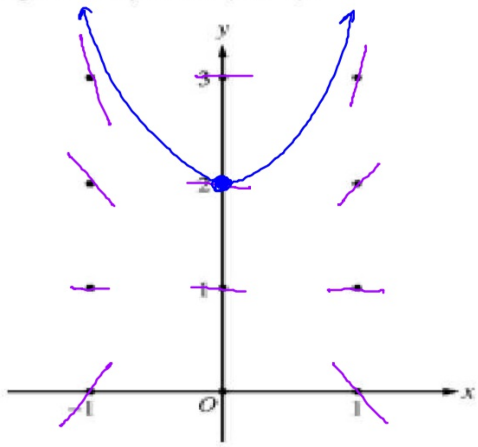
$$\boxed{w = 2000e^{\frac{1}{20}t} + 200}$$

Consider the differential equation $\boxed{\dfrac{dy}{dx} = x^3(y-1)}$

$y = 1 \quad \dfrac{dy}{dx} = 0$

$x = 0 \quad \dfrac{dy}{dx} = 0$

On the axis provided, sketch a slope field for the given differential equation at the twelve points indicated. Then sketch a possible solution through the point (0, 2).



	$\dfrac{dy}{dx}$
$(1,0) \rightarrow$	-1
$(1,2) \rightarrow$	1
$(1,3) \rightarrow$	2
$(-1,0) \rightarrow$	1
$(-1,2) \rightarrow$	-1
$(-1,3) \rightarrow$	-2

Consider the differential equation $\frac{dy}{dx} = x^3(y-1)$

Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = 0.

$$\begin{array}{cc} x & y \end{array}$$

$$\frac{dy}{dx} = x^3(y-1)$$

$$\frac{dy}{y-1} = \frac{x^3(y-1)}{y-1}\,dx$$

$$\int \frac{1}{y-1}\,dy = \int x^3\,dx$$

$$\ln|y-1| = \frac{1}{4}x^4 + C \quad \checkmark$$

$$\ln|-1| = C$$

$$0 = C$$

$$\ln(-y+1) = \frac{1}{4}x^4$$

$$-y+1 = e^{\frac{1}{4}x^4}$$

$$-y = e^{1/4\,x^4} - 1$$

$$\boxed{y = -e^{\frac{1}{4}x^4} + 1}$$

Consider the differential equation $\frac{dy}{dx} = x^3(y-1)$

- Use Euler's Method, starting at f(0) = 0, with 2 steps of equal size to approximate f(1). Show the computations that lead to your answer.

$(0, 0)$ $\quad m = \frac{dy}{dx} = 0$ $\quad y = 0 + 0(x-0)$ $\quad y = 0$ $\quad y(.5) = 0$

$(.5, 0)$ $\quad m = \frac{dy}{dx} = \left(\frac{1}{2}\right)^3(-1) = -\frac{1}{8}$ $\quad y = 0 - \frac{1}{8}\left(x - \frac{1}{2}\right)$ $\quad \boxed{y(1) = -\frac{1}{8}\left(\frac{1}{2}\right) = -\frac{1}{16}}$