

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Waits and Kennedy
Chapter 3: Derivatives 3.9: Derivatives of Exponential and Logarithmic Functions

What you'll Learn About
 How to take the derivative of exponential and logarithmic functions

$$y = x^5$$

$$\frac{dy}{dx} = 5x^4$$

$$e = 2.718\ldots$$

$$\ln_e e = 1$$

$$\ln_e e^i = 1$$

$$y' =$$

$$A) \quad y = 5^x$$

$$y = 5^x$$

$$y' = 5^x (\ln 5) \cdot 1$$

$$C) \quad y = 5^{\sin x}$$

$$\frac{dy}{dx} = 5^{\sin x} \cdot \ln 5 \cdot \cos x$$

$$E) \quad y = e^x$$

$$\frac{dy}{dx} = e^x \cdot \ln e \cdot 1$$

$$\boxed{\frac{dy}{dx} = e^x}$$

$$G) \quad y = (5e)^{5x}$$

$$y' = (5e)^{5x} \cdot \ln(5e) \cdot 5$$

$$I) \quad y = x^3 e^{4x} - x^4 e^{2x}$$

$$y = x^3 e^{4x} - x^4 e^{2x}$$

$$y' = x^3 (e^{4x} \cdot \ln e \cdot 4) + e^{4x} \cdot 3x^2 - \left[x^4 e^{2x} \cdot \ln e \cdot 2 + e^{2x} \cdot 4x^3 \right]$$

$$B) \quad y = 7^{x^2}$$

$$y' = 7^{x^2} (\ln 7) \cdot 2x$$

$$D) \quad y = 6^{\arctan x^3}$$

$$\frac{dy}{dx} = 6^{\arctan(x^3)} \cdot (\ln 6) \cdot \frac{1}{1 + (x^3)^2} \cdot 3x^2$$

$$F) \quad y = 5e^{5x}$$

$$y' = 5e^{5x} \cdot \ln e \cdot 5$$

$$y' = 25e^{5x}$$

$$H) \quad y = e^{-\frac{3}{4}x}$$

$$y' = e^{-\frac{3}{4}x} \cdot \ln e \cdot -\frac{3}{4}$$

$$y' = -\frac{3}{4}e^{-\frac{3}{4}x}$$

~~B) $y = 7^{x^2}$~~

$$y = \log_5(x^3)$$

$$5^y = x^3$$

$$y^1 = \frac{-4x^{-2}}{4x^{-1} \cdot \ln 5} = \frac{-x}{x^2 \ln 5} = \frac{-1}{x \ln 5}$$

$$y = \ln_e x$$

$$y = \log_{10} x$$

$$y = \ln^4 x$$

$$y = x^3 \ln(x^2) - \ln(\ln(\arcsin x))$$

$$y^1 = x^3 \cdot \frac{2x}{x^2 \cdot \ln e} + \ln(x^2) \cdot 3x^2 - \frac{1}{\ln(\arcsin x) \cdot \ln e} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$y^1 = 2x^2 + 3x^2 \ln(x^2)$$

$$A) \quad y = \log_5(x^3)$$

$$y^1 = \frac{1}{x^3 \cdot \ln 5} \cdot 3x^2 = \frac{3x^2}{x^3 \ln 5} = \frac{3}{x \ln 5}$$

$$B) \quad y = \log_6 \sqrt[3]{x} = \log_6(x^{1/3})$$

$$y^1 = \frac{1}{x^{1/3} \cdot \ln 6} \cdot \frac{1}{3} x^{-2/3} = \frac{1}{x^{1/3} \ln 6 \cdot 3 \cdot x^{2/3}} = \boxed{\frac{1}{3x \ln 6}}$$

$$C) \quad y = \log_5\left(\frac{4}{x}\right)$$

$$y = \log_5(4x^{-1})$$

$$E) \quad y = \ln(x)$$

$$y^1 = \frac{1}{x \ln e} \cdot 1$$

$$\boxed{y^1 = \frac{1}{x}}$$

$$G) \quad y = (\ln x)^4$$

$$y = (\ln(x))^4$$

$$y^1 = 4(\ln x)^3 \cdot \frac{1}{x \ln e} \cdot 1 = \frac{4(\ln x)^3}{x}$$

$$I) \quad y = x^3 \ln(x^2) - \ln(\ln(\arcsin x))$$

$$D) \quad y = \frac{5}{\log_7(x^2)} = 5 \left(\log_7(x^2) \right)^{-1}$$

$$y^1 = -5(\log_7(x^2))^{-2} \cdot \frac{2x}{x^2 \ln 7} = \frac{-10}{x \ln 7 (\log_7 x^2)^2}$$

$$F) \quad y = \ln(x^4)$$

$$\boxed{y^1 = \frac{4x^3}{x^4 \cdot \ln e} = \frac{4}{x}}$$

$$\left\{ \begin{array}{l} y = \ln(x^4) \\ y^1 = 4 \\ y = \ln(x^20) \\ y^1 = 20 \end{array} \right.$$

$$H) \quad y = \ln\left(\frac{5}{x}\right)$$

$$y = \ln(5x^{-1})$$

$$y^1 = \frac{-5x^{-2}}{(5x^{-1}) \cdot \ln e} = \frac{-x}{x^2} = \boxed{\frac{-1}{x}}$$