

Calculus II

Section 13.4 – Derivatives of Parametric Equations (FDWK)

Find dy/dx in terms of t without eliminating the parameter.

1. $x = 4\sin t, y = 2\cos t$

2. $x = \cos t, y = \sqrt{3}\cos t$

3. $x = -\sqrt{t+1}, y = \sqrt{3t}$

4. $x = \frac{1}{t}, y = -2 + \ln t$

5. $x = t^2 - 3t, y = t^3$

6. $x = t^2 + t, y = t^2 - t$

Find the points at which the tangent to the curve is (a) horizontal, (b) vertical.

7. $x = \sec t, y = \tan t$

8. $x = 2 + \cos t, y = -1 + \sin t$

9. $x = 2 - t$, $y = t^3 - 4t$

10. $x = e^{x^2}$, $y = xe^x$

Find the equation of the tangent to the given curve at the given point.

11. $x = 2\sin t$, $y = 2\cos t$, $t = 3\pi/4$

12. $x = 3t$, $y = 8t^3$, $t = -1/2$

13. $x = 2e^t$, $y = \frac{1}{3}e^{-t}$, $t = 0$

14. $x = t \ln t$, $y = \ln t$, $t = e$

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Section 13.4 - Derivatives of Parametric Equations (FDWK)

Find dy/dx in terms of t without eliminating the parameter.

1. $x = 4\sin t, y = 2\cos t$

$$\frac{dy}{dx} = \frac{2(-\sin t)}{4 \cos t} = -\frac{1}{2} \tan t$$

2. $x = \cos t, y = \sqrt{3} \cos t$

$$\frac{dy}{dx} = \frac{\sqrt{3}(-\sin t)}{-\sin t} = \sqrt{3}$$

3. $x = -\sqrt{t+1}, y = \sqrt{3t}$

$$\frac{dy}{dx} = \frac{\frac{1}{2}(3t)^{-1/2}(3)}{-\frac{1}{2}(t+1)^{-1/2}} = -\frac{3\sqrt{t+1}}{\sqrt{3t}}$$

4. $x = \frac{1}{t}, y = -2 + \ln t$

$$\frac{dy}{dx} = \frac{1/t}{-1/t^2} = -t$$

5. $x = t^2 - 3t, y = t^3$

$$\frac{dy}{dx} = \frac{3t^2}{2t-3}$$

6. $x = t^2 + t, y = t^2 - t$

$$\frac{dy}{dx} = \frac{2t-1}{2t+1}$$

Find the points at which the tangent to the curve is (a) horizontal, (b) vertical.

7. $x = \sec t, y = \tan t$

$$\frac{dy}{dx} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t}$$

a) $m=0$ if $\sec t = 0$
 $\frac{1}{\cos t} = 0$
NO VALUES

b) m ~~undefined~~ if $\tan t = 0$
 $\frac{\sin t}{\cos t} = 0$
 $\sin t = 0$
 $t = 0 + \pi k$

8. $x = 2 + \cos t, y = -1 + \sin t$

$$\frac{dy}{dx} = \frac{\cos t}{-\sin t}$$

a) $m=0$ if $\cos t = 0 \Rightarrow t = \pi/2 + \pi k$

b) m ~~undefined~~ if $\sin t = 0 \Rightarrow t = 0 + \pi k$

$$9. x = 2 - t, y = t^3 - 4t$$

$$\frac{dy}{dx} = \frac{3t^2 - 4}{-1}$$

$$a) m=0 \text{ if } 3t^2 - 4 = 0 \\ t^2 = 4/3 \Rightarrow t = \pm 2/\sqrt{3}$$

$$b) m \text{ und if } -1 = 0 \\ \text{NO VALUES}$$

$$10. x = e^t, y = te^t$$

$$\frac{dy}{dx} = \frac{xe^x + e^x}{2xe^{x^2}}$$

$$a) m=0 \text{ if } xe^x + e^x = 0 \\ e^x(x+1) = 0 @ t = -1$$

$$b) m \text{ und if } 2te^{t^2} = 0 @ t = 0$$

Find the equation of the tangent to the given curve at the given point.

$$11. x = 2\sin t, y = 2\cos t, t = 3\pi/4$$

$$\left. \begin{aligned} x(3\pi/4) &= 2(\sqrt{2}/2) = \sqrt{2} \\ y(3\pi/4) &= 2(-\sqrt{2}/2) = -\sqrt{2} \end{aligned} \right\} (\sqrt{2}, -\sqrt{2})$$

$$\frac{dy}{dx} = \frac{-2\sin t}{2\cos t} = -\tan t \Big|_{t=3\pi/4} = 1$$

$$y + \sqrt{2} = 1(x - \sqrt{2})$$

$$12. x = 3t, y = 8t^3, t = -1/2$$

$$\left. \begin{aligned} x(-1/2) &= -3/2 \\ y(-1/2) &= -1 \end{aligned} \right\} (-3/2, -1)$$

$$\frac{dy}{dx} = \frac{24t^2}{3} \Big|_{t=-1/2} = \frac{24(1/4)}{3} = 2$$

$$y + 1 = 2(x + 3/2)$$

$$13. x = 2e^t, y = \frac{1}{3}e^{-t}, t = 0$$

$$\left. \begin{aligned} x(0) &= 2 \\ y(0) &= 1/3 \end{aligned} \right\} (2, 1/3)$$

$$\frac{dy}{dx} = \frac{-\frac{1}{3}e^{-t}}{2e^t} \Big|_{t=0} = \frac{-1/3}{2} = -\frac{1}{6}$$

$$y - 1/3 = -\frac{1}{6}(x - 2)$$

$$14. x = t \ln t, y = \ln t, t = e$$

$$\left. \begin{aligned} x(e) &= e \ln e = e \\ y(e) &= 1 \end{aligned} \right\} (e, 1)$$

$$\frac{dy}{dx} = \frac{1/t}{t \cdot \frac{1}{t} + \ln t} \Big|_{t=e} = \frac{1/e}{1 + \ln e} = \frac{1/e}{2} = \frac{1}{2e}$$

$$y - 1 = \frac{1}{2e}(x - e)$$