

## **Derivative Review Part 1**

**3.3,3.5,3.6,3.8,3.9**

Find the derivative of the function

p. 181 #1

$$1) \quad y = x^5 - \frac{1}{8}x^2 + \frac{1}{4}x$$

$$y' = 5x^4 - \frac{2}{8}x + \frac{1}{4}x^0$$

$$y' = 5x^4 - \frac{1}{4}x + \frac{1}{4}$$

Find the derivative of the function

p. 181 #3

3)  $y = 2\sin x \cos x$

$$y' = 2\sin x(-\cos x) + \cos x(2\sin x)$$

$$y' = -2\sin^2 x + 2\cos^2 x$$

Find the derivative of the function

p. 181 #3

3)  $y = 2(\sin x \cos x)$

$$\begin{aligned}y' &= 2[\sin x \cdot (-\cos x) + \cos x (\cos x)] \\&= 2[-\sin^2 x + \cos^2 x]\end{aligned}$$

Find the derivative of the function

p. 181 #4

$$4) \quad y = \frac{2x+1}{(2x-1)}$$

$$\frac{dy}{dx} = \frac{(2x-1) \cdot 2 - (2x+1) \cdot 2}{(2x-1)^2}$$

$$4x-2 - 4x-2$$

$$\boxed{\frac{-4}{(2x-1)^2}}$$

Find the derivative of the function

p. 124 #4

$$2) \quad y = \frac{x^3}{3} - x \quad \left. \begin{array}{l} y = \frac{x^3}{3} \\ y' = \underline{\frac{3(3x^2)}{3^2} - x^3(0)} \\ y' = \frac{9x^2}{9} = x^2 \end{array} \right\}$$
$$\begin{aligned} y &= \frac{1}{3}x^3 - x \\ y' &= x^2 - 1 \end{aligned}$$

Horizontal Tangents

## Find the derivative of the function

p. 124 #7

7)  $y = x^3 - 2x^2 + x + 1$  ③ solve

$$y' = 3x^2 - 4x + 1$$

$$0 = 3x^2 - 4x + 1$$

$$0 = (3x-1)(x-1)$$

$$x = \frac{1}{3} \quad x = 1$$

① Find  $y'$

② Set  $y' = 0$

③ solve

④ Plug x-value

back into original

Find the derivative of the function

p. 124 #13

$$13) \quad y = (x+1)(x^2 + 1)$$

$$y' = (x+1)(2x) + (x^2+1)(1)$$

$$y' = 2x^2 + 2x + x^2 + 1$$

$$y' = 3x^2 + 2x + 1$$

Find the derivative of the function

p. 124 #13

$$13) \quad y = (x+1)(\overbrace{x^2+1}^{\text{circled}})$$

$$y = x^3 + x + 1x^2 + 1$$

$$y = x^3 + x^2 + x + 1$$

$$y' = 3x^2 + 2x + 1$$

Find the derivative of the function

p. 124 #17

17)  $y = \frac{2x+5}{3x-2}$

$\frac{dy}{dx} = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2}$

$\rightarrow \frac{6x-4 - 6x-15}{(3x-2)^2}$

$y' = \frac{-19}{(3x-2)^2}$

Find the derivative of the function

p. 124 #29

$$29) \quad y = 4x^{-2} - 8x + 1$$

$$y' = -8x^{-3} - 8$$

$$y' = -\frac{8}{x^3} - 8$$

Find the derivative of the function

p. 124 #30

$$30) \quad y = \frac{x^{-4}}{4} - \frac{x^{-3}}{3} + \frac{x^{-2}}{2} - x^{-1} + 1$$

$$y = \frac{1}{4}x^{-4} - \frac{1}{3}x^{-3} + \frac{1}{2}x^{-2} - x^{-1} + 1$$

$$y' = -1x^{-5} + 1x^{-4} - 1x^{-3} + 1x^{-2}$$

$$y' = -\frac{1}{x^5} + \frac{1}{x^4} - \frac{1}{x^3} + \frac{1}{x^2}$$

a) Find all points where  $f$  has horizontal tangents

p. 126 Quick Quiz 4

$$f(x) = x^4 - 4x^2$$

$$f'(x) = 4x^3 - 8x$$

$$0 = 4x^3 - 8x$$

$$0 = 4x(x^2 - 2)$$

$$4x = 0 \quad x^2 - 2 = 0$$

$$x = 0 \quad x = \pm\sqrt{2}$$

a) Find an equation of the tangent line at  $x = 1$ .

p. 126 Quick Quiz 4

(1, -3)

$$f(x) = x^4 - 4x^2 \quad f(1) = 1^4 - 4(1)^2 = 1 - 4 = -3$$

$$f'(x) = 4x^3 - 8x$$

$$f'(1) = 4 - 8 = -4$$

$$y = -3 - 4(x-1)$$

b) Find an equation of the normal line at  $x = 1$ .

p. 126 Quick Quiz 4

Tangent Line = -4

$$f(x) = x^4 - 4x^2$$

Normal Line =  $\frac{1}{4}$

$$y = -3 + \frac{1}{4}(x-1)$$

Find  $\frac{dy}{dx}$

p. 146 4

$$y = x \sec x$$

$$y' = x (\sec x \tan x) + \sec x (1)$$

Find  $\frac{dy}{dx}$

p. 146 6

$$y = 3x + x \tan x$$

$$y' = 3 + x(\sec^2 x) + \tan x(1)$$

Find  $\frac{dy}{dx}$

p. 146 8

$$y = \frac{x}{1 + \cos x}$$

$$y' = \frac{(1 + \cos x)(1) - x(-\sin x)}{(1 + \cos x)^2}$$

$$\frac{1 + \cos x + x \sin x}{(1 + \cos x)^2}$$

Find  $\frac{dy}{dx}$

p. 146 8

$$y = \frac{x}{1 + \cos x}$$

$$y = x(1 + \cos x)^{-1}$$

$$y' = x \left[ -1(1 + \cos x)^{-2} \cdot (-\sin x) \right] + (1 + \cos x)^{-1}(1)$$

Find  $\frac{dy}{dx}$

p. 153 17

$$y = \sin^3 x \tan 4x$$

$$y = [\sin x]^3 + \tan(4x)$$

$$y' = [\sin x]^3 \cdot \sec^2(4x) \cdot 4 + \tan(4x) \cdot 3[\sin x]^2 \cdot \cos x \cdot 1$$

Find  $\frac{dy}{dx}$

p. 153 19

$$y = \frac{3}{\sqrt{2x+1}} = \frac{3}{(2x+1)^{1/2}} = 3(2x+1)^{-1/2}$$

$$y' = -\frac{3}{2}(2x+1)^{-3/2} \cdot \frac{2}{1} = -3(2x+1)^{-3/2}$$

Find  $\frac{dy}{dx}$

p. 153 21

$$y = \sin^2(3x - 2) = [\sin(3x-2)]^2$$

$$y' = 2[\sin(3x-2)] \cdot \cos(3x-2) \cdot 3$$

$$y' = 6 \sin(3x-2) \cos(3x-2)$$

Find  $\frac{dy}{dx}$

p. 153 24

$$y = \sqrt{\tan 5x} = (\tan 5x)^{\frac{1}{2}}$$

$$y' = \frac{1}{2} (\tan 5x)^{-\frac{1}{2}} \cdot \sec^2(5x) \cdot 5$$

$$y' = \frac{5 \sec^2(5x)}{2\sqrt{\tan 5x}}$$

Extra Practice

Find  $\frac{dy}{dx}$

$$y = \sqrt{\tan \sqrt{5x}} = (\tan(5x)^{\frac{1}{2}})^{\frac{1}{2}}$$
$$y' = \frac{1}{2} (\tan(5x)^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \sec^2(5x)^{\frac{1}{2}} \cdot \frac{1}{2}(5x)^{-\frac{1}{2}} \cdot 5$$

Find the derivative of  $y$  with respect to the given variable

p. 156 Quick Quiz #1

$$y = \sin^4(3x) = [\sin(3x)]^4$$

$$y' = 4[\sin(3x)]^3 \cdot \cos(3x) \cdot 3$$

$$y' = 12[\sin(3x)]^3 \cos(3x)$$

Find the derivative of  $y$  with respect to the given variable

p. 170 #3  $\frac{d}{dx} \{t\sqrt{2}\} = \sqrt{2}$        $\frac{d}{dx} \{2t\} = 2$

$$y = \sin^{-1} \sqrt{2}t = \arcsin(t\sqrt{2})$$

$$y' = \frac{1}{\sqrt{1 - (t\sqrt{2})^2}} \cdot \sqrt{2} = \frac{\sqrt{2}}{\sqrt{1 - 2t^2}}$$

Find the derivative of  $y$  with respect to the given variable

Extra Practice

$$y = \sin^{-1} \sqrt{2t} = \arcsin(2t)^{\frac{1}{2}}$$
$$y' = \frac{1}{\sqrt{1 - (\sqrt{2t})^2}} \cdot \frac{1}{2} (2t)^{-\frac{1}{2}} \cdot 2 = \frac{1}{\sqrt{2t} \sqrt{1-2t}}$$

Find the derivative of  $y$  with respect to the given variable

p. 170 #21

$$\frac{x}{\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$$

$$y = \tan^{-1} \sqrt{x^2-1} + \csc^{-1} x$$

$$y = \arctan(x^2-1)^{-1/2} + \arccsc x$$

$$y' = \frac{1}{1+(\sqrt{x^2-1})^2} \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x + \frac{-1}{|x|\sqrt{x^2-1}}$$

Find  $\frac{dy}{dx}$

p. 178 #8

$$y = (x^2 e^x) - (x e^x)$$

$$y' = x^2 e^x + e^x(2x) - [x e^x + e^x(1)]$$

$$= x^2 e^x + 2x e^x - (x e^x - e^x)$$

$$= x^2 e^x + x e^x - e^x$$

Find  $\frac{dy}{dx}$

p. 178 #13

$$y = 3^{\csc x}$$

$$\frac{dy}{dx} = 3^{\csc x} \cdot \ln 3 \cdot (-\csc x \cot x)$$

Find  $\frac{dy}{dx}$

p. 178 #16

$$y = (\ln x)^2$$

$$\frac{dy}{dx} = 2(\ln x) \cdot \frac{1}{x} = \frac{2}{x} \ln x$$

Find  $\frac{dy}{dx}$

p. 178 #19

$$y = \ln(\ln x)$$

$$\frac{dy}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

Find  $\frac{dy}{dx}$

p. 178 #21

$$y = \log_4(x^2)$$

$$\frac{dy}{dx} = \frac{1}{x^2 \ln 4} \cdot 2x = \frac{2}{x \ln 4}$$

Find the derivative of the function

p. 181 #8

$$y = x\sqrt{2x+1} = x(2x+1)^{\frac{1}{2}}$$
$$\frac{dy}{dx} = x \left[ \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2 \right] + \sqrt{2x+1} (1)$$
$$= \frac{x}{\sqrt{2x+1}} + \sqrt{2x+1}$$

Find the derivative of the function

p. 181 #11

$$y = (x^2) \csc x$$

$$\frac{dy}{dx} = x^2 (-\csc x \cot x) + \csc x (2x)$$

Find the derivative of the function

p. 181 #17

$$y = \ln(\arccos x)$$

$$y' = \frac{1}{\arccos x} \cdot \frac{-1}{\sqrt{1-x^2}}$$

Find the derivative of the function

p. 181 #24

$$y = \arcsin \sqrt{1-x^2} = \arcsin (1-x^2)^{1/2}$$

$$y' = \frac{1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2} (1-x^2)^{-1/2} \cdot -2x$$

$$y' = \frac{-1}{\sqrt{x^2} \sqrt{1-x^2}}$$

Find the derivative of the function

p. 182 #30

$$y = \left( \frac{1 + \sin x}{1 - \cos x} \right)^2$$
$$y' = 2 \left( \frac{1 + \sin x}{1 - \cos x} \right) \cdot \left[ \frac{(1 - \cos x)(\cos x) - (1 + \sin x)(\sin x)}{(1 - \cos x)^2} \right]$$

p. 183 #67d

Suppose that the functions  $f$  and  $g$  and their first derivatives have the following values at  $x = -1$  and  $x = 0$ .  
Find the first derivative of the following

$$f(g(x))$$

$$\begin{aligned}\frac{d}{dx} \left[ f(g(x)) \right] &= f'(g(x)) \cdot g'(x) \\ &= f'(g(-1)) \cdot g'(-1) \\ &= f'(-1) \cdot g'(-1) = 2(1) = 2\end{aligned}$$

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

$$(3, 15) \quad m = -8 \quad \left\{ \begin{array}{l} (15, 3) \quad m = -\frac{1}{8} \end{array} \right.$$

1. Let  $f$  be a differentiable function such that  
 $f(3) = 15$ ,  $f(6) = 3$ ,  $f'(3) = -8$  and  
 $f'(6) = -2$ .

The function  $g$  is differentiable and  
 $g(x) = f^{-1}(x)$  for all  $x$ . What is the value  
of  $g'(15)$ ?

- a)  $-1/2$
- b)  $-1/8$
- c)  $1/6$
- d)  $1/3$
- e) The value of  $g'(15)$  cannot be determined

$$(2, -3) \quad m = \frac{4}{3} \quad \left\{ \begin{array}{l} (-3, 2) \quad m = \frac{3}{4} \end{array} \right.$$

4. If  $f(2) = -3$ ,  $f'(2) = \frac{4}{3}$ , and  $g(x) = f^{-1}(x)$ ,  
what is the equation of the tangent line to  $g(x)$   
at  $x = -3$ ?

A)  $y - 2 = \frac{-3}{4}(x + 3)$

B)  $y + 2 = \frac{-3}{4}(x - 3)$

C)  $y - 2 = \frac{3}{4}(x + 3)$

D)  $y + 3 = \frac{3}{4}(x - 2)$

E)  $y - 2 = \frac{4}{3}(x + 3)$

p. 92 #11

$$y = 1 - \frac{1}{x-2}$$

Find the equation of the tangent and normal line to  
the curve at the given point  $y = 1 + \frac{1}{x-2}$

$$y = \frac{1}{x-1} \quad \text{at } x = 2 \quad y = \frac{1}{2-1} = 1 \quad (2, 1)$$

$$y = (x-1)^{-1}$$

$$y' = - (x-1)^{-2} = \frac{-1}{(x-1)^2} \quad y'(2) = \frac{-1}{(2-1)^2} = -1$$

p. 92 #11

$$y = -1 - 3(x-0)$$

Find the equation of the ~~tangent~~ and ~~normal~~ line to  
the curve at the given point

$$y = -1 + \frac{1}{3}(x-0)$$

$$y = x^2 - 3x - 1 \quad \text{at } x = 0 \quad (0, -1)$$

$$y' = 2x - 3$$

$$y'(0) = -3$$

p. 182 #51

Find the equation for the line tangent to the curve at the given value of  $t$

$$x = 3 \sec t \quad y = 5 \tan t \quad \text{at } t = \frac{\pi}{6}$$

$$x = 3 \sec \frac{\pi}{6} \quad y = 5 \tan \frac{\pi}{6}$$

$$x = 3 \left( \frac{2}{\sqrt{3}} \right) \quad y = 5 \left( \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right)$$

$$x = \frac{6}{\sqrt{3}} \quad y = \frac{5}{\sqrt{3}}$$

p. 182 #51

Find the equation for the line tangent to the curve at the given value of  $t$

$$y = \frac{5}{\sqrt{3}} + \frac{10}{3} \left( x - \frac{6}{\sqrt{3}} \right)$$

$$x = 3 \sec t \quad y = 5 \tan t \quad \text{at } t = \frac{\pi}{6}$$

$$\frac{dy}{dt} = 5 \sec^2 t$$

$$\frac{dx}{dt} = 3 \sec t \tan t$$

$$\frac{dy}{dx} = \frac{5 \sec^2 t}{3 \sec t \tan t} = \frac{5 \sec t}{3 \tan t} = \frac{5 \left( \frac{2}{\sqrt{3}} \right)}{3 \left( \frac{1}{\sqrt{3}} \right)} = \frac{10}{3}$$

p. 535 #25

Find the points at which the tangent line to the curve  
is horizontal and/or vertical

$$\frac{dy}{dt} = 0 \quad \frac{dx}{dt} = 0$$

$$x = 2 - t \quad y = t^3 - 4t$$

$$\frac{dx}{dt} = -1 \quad \frac{dy}{dt} = 3t^2 - 4$$

No Vertical Tangents

$$0 = 3t^2 - 4$$
$$\frac{4}{3} = t^2 \quad t = \pm \sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}}$$

1. A curve C is defined by the parametric equations  $x = t^2 - 4t + 1$  and  $y = t^3$ . Find the equation of the line tangent to the graph of C at the point  $(1, 64)$ ?  $x = t^2 - 4t + 1$

$$\frac{dy}{dx} = \frac{3t^2}{2t-4} =$$

$$\left. \frac{dy}{dx} \right|_{t=4} = \frac{3(4)^2}{2(4)-4} = \frac{48}{8} = 6$$

$$\begin{aligned} 1 &= t^2 - 4t + 1 \\ 0 &= t^2 - 4t \\ 0 &= t(t-4) \quad t=0(4) \\ y &= t^3 \\ 64 &= t^3 \quad t=4 \end{aligned}$$

2. If  $f(x) = (\ln x)^2$ , then,  $f''(e^2) =$

$$f'(x) = \frac{2 \ln x}{x}$$

$$f'(x) = \frac{x \left[ \frac{2}{x} \right] - 2 \ln x \cdot 1}{x^2}$$

$$f''(x) = \frac{2 - 2 \ln x}{x^2}$$

$$f''(e^2) = \frac{2 - 2 \boxed{\ln e^2}}{(e^2)^2} \rightarrow 2$$

$$= \frac{2-4}{e^4} = \frac{-2}{e^4}$$

1. A curve C is defined by the parametric equations  $x = t^2 - 4t + 1$  and  $y = t^3$ . Find the equation of the line tangent to the graph of C at the point  $(-2, 27)$ ?

$$\frac{dy}{dx} = \frac{3t^2}{2t-4}$$

$$27 = t^3$$
$$t = 3$$

$$\left. \frac{dy}{dx} \right|_{t=3} = \frac{3(3)^2}{2(3)-4} = \frac{27}{2}$$

$$y = 27 + \frac{27}{2}(x+2)$$

2. Let  $h$  be a differentiable function, defined by  $f(x) = h(x^3 - 4)$ . Find  $f'(3)$ .

$$f(x) = h(x^3 - 4)$$

$$f'(3) = 27h'(23)$$

$$f'(x) = h'(x^3 - 4) \cdot 3x^2$$

$$f'(3) = h'(3^3 - 4) \cdot 3(3)^2$$

$$f'(3) = h'(23) \cdot 27$$