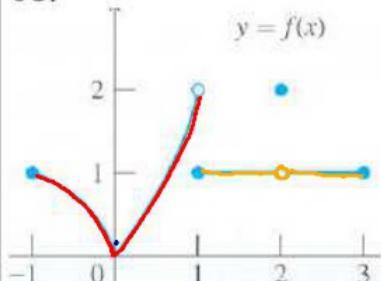


CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Waits and Kennedy  
 Chapter 2: Limits and Continuity      2.1: Limits at a point pg. 59-69

What you'll Learn About

- One-Sided and Two Sided Limits
- Properties of Limits

38.



$$\lim_{x \rightarrow 1^-} f(x)$$

\* As  $x$  approaches 1 from the left side the limit of  $f(x)$  is y-value

\* The limit of  $f(x)$  as  $x$  approaches 1 from the left side is y-value

$$\lim_{x \rightarrow 1} f(x) =$$

approaching from the left and right  
 ↑ (1,   )  
 point

a)  $\lim_{x \rightarrow 1^-} f(x) = 2$

b)  $\lim_{x \rightarrow 1^+} f(x) = 1$

c)  $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

d)  $f(1) = 1$

e)  $\lim_{x \rightarrow 0^-} f(x) = 0$

f)  $\lim_{x \rightarrow 0^+} f(x) = 0$

g)  $\lim_{x \rightarrow 0} f(x) = 0$

h)  $f(0) = 0$

i)  $\lim_{x \rightarrow 1^+} f(x) = 1$

j)  $\lim_{x \rightarrow 3^-} f(x) = 1$

k)  $\lim_{x \rightarrow 2} f(x) = 1$

True/False

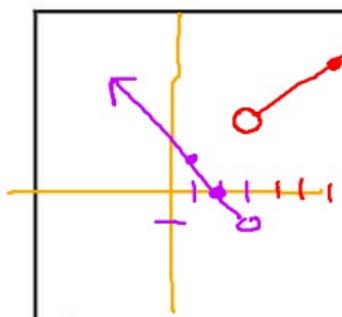
l)  $\lim_{x \rightarrow c} f(x)$  exists at every  $c$  between  $(-1, 1)$  X X

m)  $\lim_{x \rightarrow c} f(x)$  exists at every  $c$  between  $(1, 3)$  True

n)  $\lim_{x \rightarrow c} f(x)$  exists at every  $c$  between  $(-1, 3)$

False  
 $x=1$  Jump Limit DNE

definition  
 & continuity



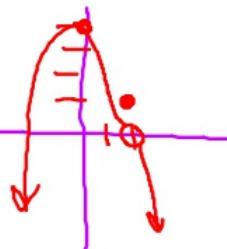
Determine the limits for the piecewise function given below

51A.  $f(x) = \begin{cases} 2-x & x < 3 \\ \frac{x}{3} + 1 & x > 3 \end{cases}$  Left   
 Right

a)  $\lim_{x \rightarrow 3^-} f(x) = 2-3 = -1$  b)  $\lim_{x \rightarrow 3^+} f(x) = \frac{3}{3} + 1 = 2$  c)  $\lim_{x \rightarrow 3} f(x) = \text{DNE}$  d)  $f(3) = \text{DNE}$

$$\begin{array}{|c|c|} \hline x & y = 2-x \\ \hline 3 & -1 \\ 2 & 0 \\ 1 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline x & y = \frac{x}{3} + 1 \\ \hline 3 & 2 \\ 2 & 1 \\ 1 & \frac{4}{3} \\ \hline \end{array}$$



$$f(x) = \frac{1}{x-2}$$

$$\begin{aligned} f(1.9) &= \frac{1}{1.9-2} \\ &= \frac{1}{-0.1} \\ &= -10 \end{aligned}$$

54A.  $f(x) = \begin{cases} 4-x^2 & x \neq 2 \\ 1 & x = 2 \end{cases}$

a)  $\lim_{x \rightarrow 2^-} f(x) = 4-2^2 = 0$  b)  $\lim_{x \rightarrow 2^+} f(x) = 4-2^2 = 0$  c)  $\lim_{x \rightarrow 2} f(x) = 0$  d)  $f(2) = 1$

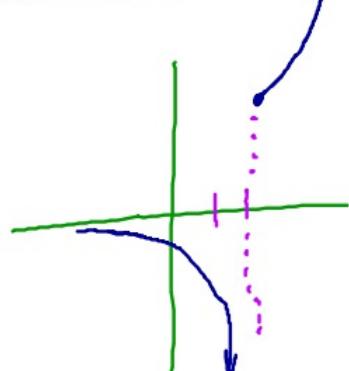
53A.  $f(x) = \begin{cases} \frac{1}{x-2} & x < 2 \\ x^3 & x > 2 \end{cases}$  Left

a)  $\lim_{x \rightarrow 2^-} f(x) = \frac{1}{2-2} = \infty$  b)  $\lim_{x \rightarrow 2^+} f(x) = 2^3 = 8$  c)  $\lim_{x \rightarrow 2} f(x) = \text{DNE}$  d)  $f(2) = 2^3 = 8$

$$= \frac{1}{0}$$

Vertical Asy

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$



Determine the Limit by substitution

$$7A) \lim_{x \rightarrow -1} 2x^2(5x+2) = \text{y-value}$$

$$\begin{aligned} & 2(-1)^2(5 \cdot -1 + 2) \\ & 2(-3) = -6 \end{aligned}$$

$$13A) \lim_{x \rightarrow 30} (x-3)^{1/3} = \frac{(30-3)}{\sqrt[3]{27}}^{1/3} = 3$$

Hole at  
 $(4, \frac{5}{8})$



Determine the limit algebraically and support graphically.

$$20A) \lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{x^2 - 16} = \frac{4^2 - 3(4) - 4}{4^2 - 16} = \frac{16 - 12 - 4}{16 - 16} = \frac{0}{0}$$

$$\begin{aligned} & \boxed{\lim_{x \rightarrow 4}} \frac{x^2 - 3x - 4}{x^2 - 16} = \frac{(x+1)(x-4)}{(x+4)(x-4)} = \frac{x+1}{x+4} = \frac{5}{8} \end{aligned}$$

Do More Algebra  
Factor

$$22A) \lim_{x \rightarrow 0} \frac{x+3}{x} =$$

Determine the limit by substitution

$$A) \lim_{x \rightarrow 1^-} \frac{5x}{|x|}$$

$$B) \lim_{x \rightarrow 1^+} \frac{5x}{|x|}$$

Determine the limit by substitution and support graphically.

$$30A) \lim_{x \rightarrow 4} \frac{x^2 - 4}{x^2 - 16} =$$

Use properties of limits to determine each limit

Assume that  $\lim_{x \rightarrow 1} f(x) = 10$  and  $\lim_{x \rightarrow 1} g(x) = 5$

$$A) \lim_{x \rightarrow 1} (f(x) + 3) = 10 + 3 \\ = 13$$

$$B) \lim_{x \rightarrow 1} (xg(x)) = (1 \cdot g(1)) = 1 \cdot 5 = 5$$

$$C) \lim_{x \rightarrow 1} (f^2(x)) = f^2(1) \\ = [f(1)]^2 \\ = 10^2 \\ = 100$$

$$D) \lim_{x \rightarrow 1} \frac{f(x)}{g(x) + 2} = \frac{f(1)}{g(1) + 2} = \frac{10}{5 + 2} = \frac{10}{7}$$