10.2: Vectors

What you'll Learn About

- Calculus using Vectors

$\langle 4\sin t, 2\cos t \rangle$

$x = 4\sin t$

$y = 2\cos t$

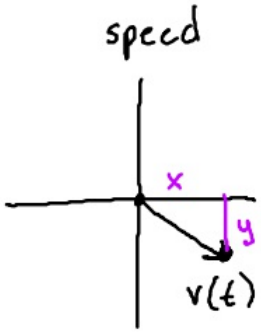
A particle moves in an elliptical path so that its position at any time $t \geq 0$ is given by $\langle 4\sin t, 2\cos t \rangle$

a) Find the velocity and acceleration vectors.

$s(t) = \langle 4\sin t, 2\cos t \rangle$

$v(t) = \langle 4\cos t, -2\sin t \rangle$

$a(t) = \langle -4\sin t, -2\cos t \rangle$



speed = $|v(t)|$

- Distance from 0

b) Find the velocity and acceleration vectors and the speed at $t = \frac{\pi}{4}$.

$v\left(\frac{\pi}{4}\right) = \left\langle 4\cos\frac{\pi}{4}, -2\sin\frac{\pi}{4} \right\rangle = \left\langle \frac{4\sqrt{2}}{2}, \frac{-2\sqrt{2}}{2} \right\rangle = \langle 2\sqrt{2}, -\sqrt{2} \rangle$

$a\left(\frac{\pi}{4}\right) = \left\langle -4\sin\frac{\pi}{4}, -2\cos\frac{\pi}{4} \right\rangle = \langle -2\sqrt{2}, -\sqrt{2} \rangle$

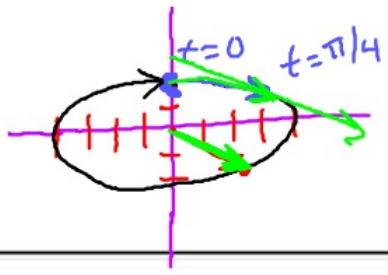
$\text{Speed} = |v(t)| = \sqrt{[v_x(t)]^2 + [v_y(t)]^2}$

$= \sqrt{(2\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{8+2} = \sqrt{10}$

$\langle 4\sin t, 2\cos t \rangle$

$4\sin\frac{\pi}{4}, 2\cos\frac{\pi}{4}$

c) Sketch the path of the particle and show the velocity vector at $t = \frac{\pi}{4}$



The velocity v(t) of a particle moving in the plane is given, along with the position of the particle at time t = 0.

39) $v(t)=\left\langle \frac{1}{t+1},(t+2)^{-2} \right\rangle$ (5, -2) $t=0$ $x=5$ $y=-2$

a) Find the position of the particle at time t = 3.

$$x(3)=5+\int_0^3 \frac{1}{t+1}\,dt \qquad y(3)=-2+\int_0^3 (t+2)^{-2}\,dt$$

$$x(3)=5+\ln|t+1|\Big]_0^3 \qquad y(3)=-2+-(t+2)^{-1}\Big]_0^3$$

$$x(3)=5+[\ln 4-\ln 1] \qquad =-2+\left[-\frac{1}{t+2}\right]_0^3$$

$$x(3)=5+\ln 4$$

$$y(3)=-2+\left[-\frac{1}{5}-\left(-\frac{1}{2}\right)\right]$$

$$=-2+\left[-\frac{1}{5}+\frac{1}{2}\right]$$

$$=-2+3/10$$

$$y(3)=-17/10$$

$\langle 5+\ln 4, -17/10 \rangle$

b) Find the distance traveled from t = 0 to t = 3.

$v(t)=\left\langle \frac{1}{t+1},(t+2)^{-2} \right\rangle$

$$\text{Total Distance}=\int |v(t)|\,dt=\int \sqrt{[v_x(t)]^2+[v_y(t)]^2}$$

$$=\int_0^3 \sqrt{\left(\frac{1}{t+1}\right)^2+\left[(t+2)^{-2}\right]^2}$$