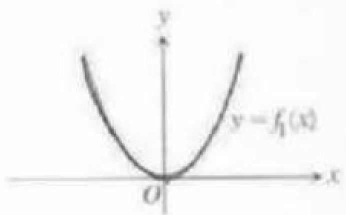
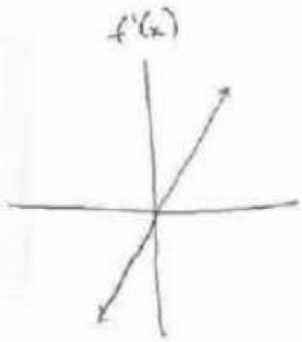
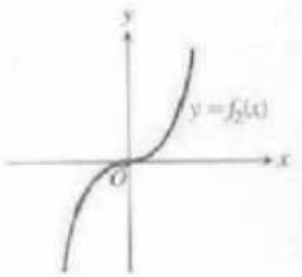
p. 105 13-16

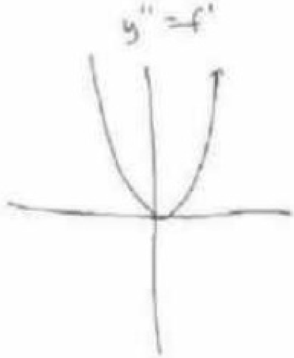
Sketch the graph of the derivative given the original function



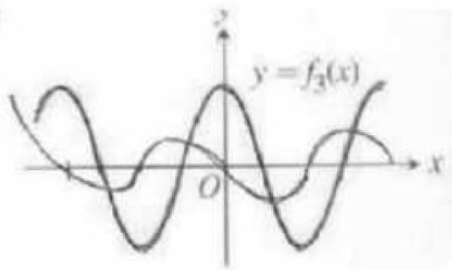


Sketch the graph of the derivative given the original function

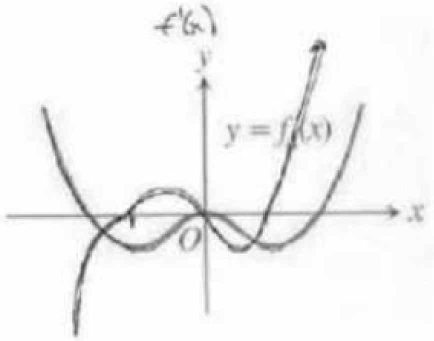




Sketch the graph of the derivative given the original function



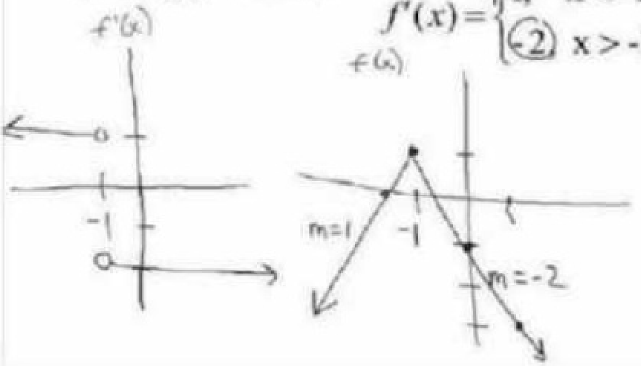
Sketch the graph of the derivative given the original function



p. 106 27

- Sketch the graph of a continuous function f with f(0) = -1 and

$$f'(x)=\begin{cases}1, & x<-1\\ -2 & x>-1\end{cases}$$

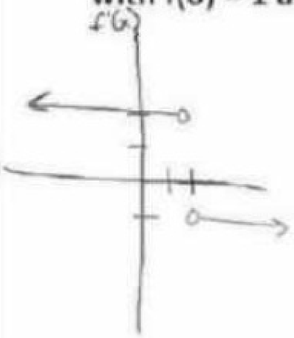


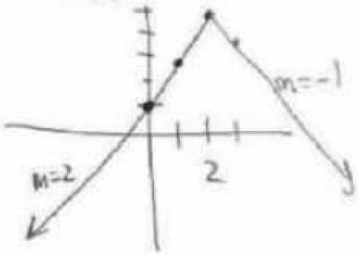
p. 106 28

- Sketch the graph of a continuous function f with f(0) = 1 and

$$f'(x) = \begin{cases} 2, & x < 2 \\ -1, & x > 2 \end{cases}$$

f(x)





p. 136 8

The number of gallons of water in a tank t minutes after the tank has started to drain is $z(t) = 200(30 - t)^2$. How fast is the water running out at the end of 10 minutes?

$z'(t) = -400(30-t)$

$z'(10) = -400(30-10)$

$z'(10) = -400(20)$

Speed $= |-8000| = 8000$ gal/min

p. 136 8

The number of gallons of water in a tank t minutes after the tank has started to drain is $z(t) = 200(30 - t)^2$. What is the average rate at which the water flows out during the first 10 minutes?

$z(0) = 200(30)^2 = 200(900) = 180000$

$z(10) = 200(20)^2 = 200(400) = 80000$

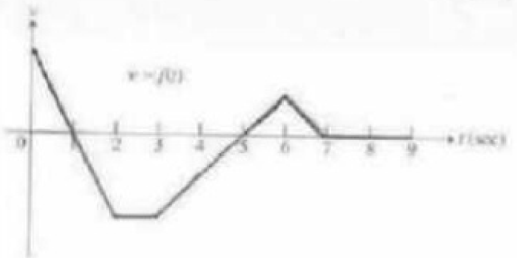
$(0, 180000)$

$(10, 80000)$

Avg. Velocity $= \dfrac{80000 - 180000}{10 - 0} = \dfrac{-100000}{10}$

$= -10000$ gal/min

p. 136 #9

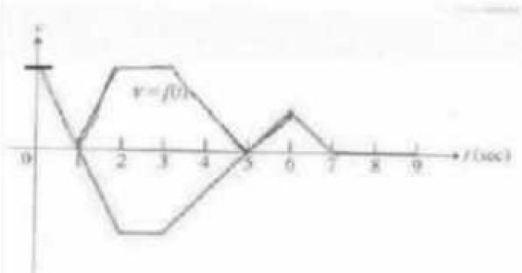


a. When does the particle move forward? Move backward?

(0,1) (5,7) forward

(1,5) backward

p. 136 #9



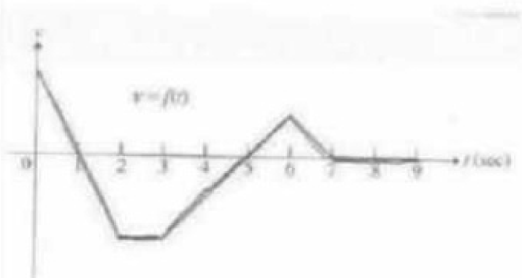
a. When does the particle speed up? Slow down?

(1,2) (5,6) speed up v(t) away from x-axis

(0,1) (3,5) (6,7) slow down

v(t) toward x-axis

p. 136 #9



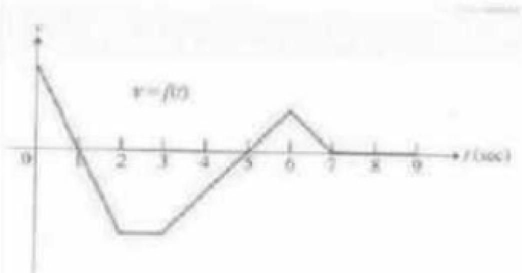
b. When is the particle's acceleration positive? Negative? Zero?

$(3,6)$ $a(t) > 0$

$(0,2)$ $(6,7)$ $a(t) < 0$

$(2,3)$ $(7,9)$ $a(t) = 0$

p. 136 #9



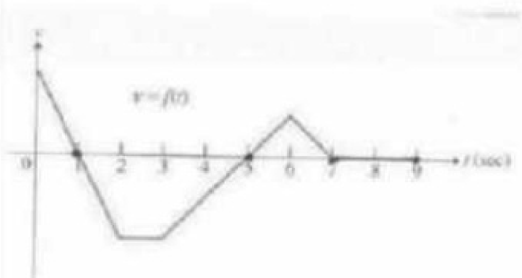
c. When does the particle move at its greatest speed?

$v(t)$ is furthest away from x-axis

$t = 0$

$(2, 3)$

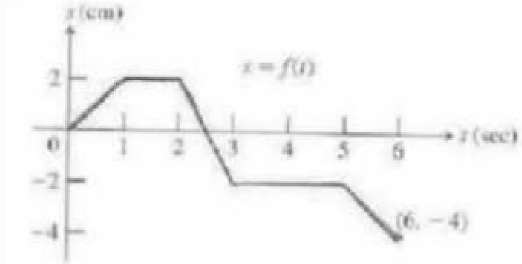
p. 136 #9



d. When is the particle at rest?

$t = 1, 5, (7, 9)$

p. 136 #10



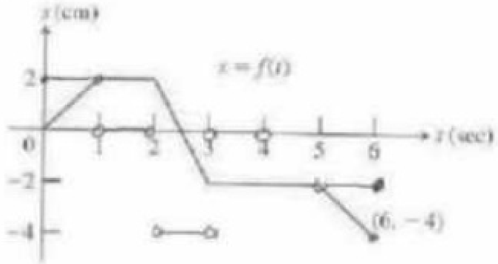
a. When is the particle moving to the left? Moving to the right? Standing Still

(0,1)

nt (1,2) (3,5) ← Standing Still

+ (2,3) (5,6)

p. 136 #10



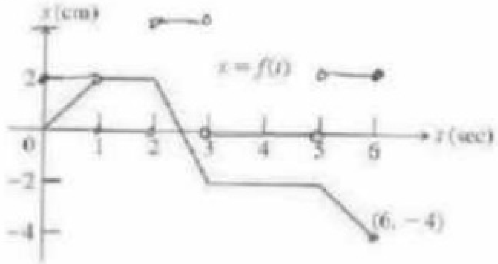
b. Graph the particles velocity.

(0,1) m=2 (3,4) m=0

(1,2) m= 0

(2,3) m= -4 (5,6) m=-2

p. 136 #10



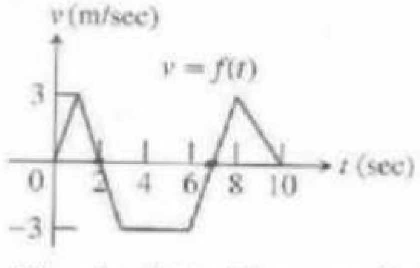
b. Graph the particles speed.

(0,1) speed = 2 (3,5) speed = 0

(1,2) speed = 0 (5,6) speed = 2

(2,3) speed = 4

p. 136 #11



a. When does the particle reverse direction?

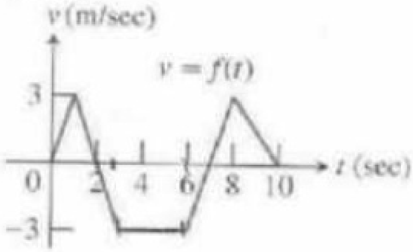
standing still $v(t)=0$ $t=0,2,7,10$

$v(t)$ changes sign

- pos to neg (above to below)
- neg to pos (below to above)

$t=2,7$

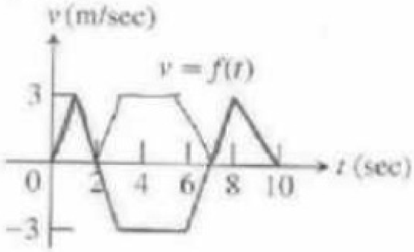
p. 136 #11



b. When is the body moving at a constant speed?

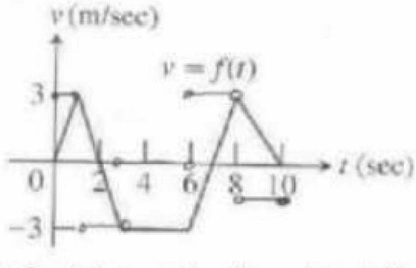
Speed is 3 from (3,6)

p. 136 #11



c. Graph the body's speed.

p. 136 #11



d. Graph the acceleration, where defined.

$(0,1)$ $a(t) = 3$ $(6,8)$ $a(t) = 3$

$(1,3)$ $a(t) = {}^{-}3$ $(8,10)$ $a(t) = -\frac{3}{2} = -1.5$

$(3,6)$ $a(t) = 0$

p. 137 19 a - e

- A particle moves along a line so that its position at any time $t > 0$ is given by the function $s(t) = t^2 - 3t + 2$, where is measured in meters and t is measured in seconds.

a) Find the displacement during the first 5 seconds.

$s(0) = 2$

$s(5) = 5^2 - 15 + 2 = 12$

$12 - 2 = 10$ meters

end start

p. 137 19 a - e

- A particle moves along a line so that its position at any time $t > 0$ is given by the function $s(t) = t^2 - 3t + 2$, where is measured in meters and t is measured in seconds.

b) Find the average velocity during the first 5 seconds.

$s(0) = 2$
$s(5) = 12$

avg velocity $= \frac{12-2}{5-0} = \frac{10}{5} = 2$ m/sec

p. 137 19 a - e

- A particle moves along a line so that its position at any time $t > 0$ is given by the function $s(t) = t^2 - 3t + 2$, where is measured in meters and t is measured in seconds.

c) Find the instantaneous velocity when $t = 4$

$v(t) = 2t - 3$

$v(4) = 8 - 3 = 5$ m/sec

p. 137 19 a - e

- A particle moves along a line so that its position at any time $t > 0$ is given by the function $s(t) = t^2 - 3t + 2$, where is measured in meters and t is measured in seconds.

d) Find the acceleration when $t = 4$

$a(t) = 2\ m/sec^2$

p. 137 19 a - e

- A particle moves along a line so that its position at any time $t > 0$ is given by the function $s(t) = t^2 - 3t + 2$, where is measured in meters and t is measured in seconds.

e) At what values does the particle change direction

0 1.5

$v(t) = 2t - 3$

$0 = 2t - 3$

$t = 1.5$

$v(1) = 2(1) - 3 < 0$ s(t) left

$v(2) = 2(2) - 3 > 0$ s(t) right

p. 137 19 a - e

- A particle moves along a line so that its position at any time $t > 0$ is given by the function $s(t) = t^2 - 3t + 2$, where is measured in meters and t is measured in seconds.

g) Describe the motion

p. 137 23

- The position of a body at time t sec is $s = t^3 - 6t^2 + 9t$ meters. Find the body's acceleration each time the velocity is zero.

$v(t) = 3t^2 - 12t + 9$

$0 = t^2 - 4t + 3$

$0 = (t-3)(t-1)$

$t = 3 \quad t = 1$

$a(t) = 6t - 12$

$a(1) = 6(1) - 12 = -6$ m/s

$a(3) = 6(3) - 12 = 6$ m/

p. 156 QQ #4

- A particle moves along a line so that its position at any time $t > 0$ is given by

$s(t) = -t^2 + t + 2$, where s is measured in meters and t is measured in seconds.

a) What is the initial position of the particle.

$s(0) = 2$

p. 156 QQ #4

- A particle moves along a line so that its position at any time $t > 0$ is given by

$s(t) = -t^2 + t + 2$, where s is measured in meters and t is measured in seconds.

b) Find the velocity of the particle at any time t.

$v(t) = -2t + 1$

p. 156 QQ #4

- A particle moves along a line so that its position at any time $t > 0$ is given by

  $s(t) = -t^2 + t + 2$, where s is measured in meters and t is measured in seconds.

c) Find the acceleration of the particle at any time t.

$v(t) = -2t + 1$

$a(t) = -2 \text{ m/sec}^2$

p. 156 QQ #4

- A particle moves along a line so that its position at any time $t > 0$ is given by $s(t) = -t^2 + t + 2$, where s is measured in meters and t is measured in seconds.

d) Find the speed of the particle at the moment when $s(t) = 0$.

$0 = -t^2 + t + 2$

$0 = t^2 - t - 2$

$0 = (t - 2)(t + 1)$

$t = 2 \quad t = -1$

$v(t) = -2t + 1$

$v(2) = -2(2) + 1$

$v(2) = -3$

$|v(2)| = 3 \text{ m/sec}$

p. 180 QQ #4

- A curve in the xy-plane is defined by $xy^2 - x^3y = 6$

a) Find dy/dx

$$x(2y)\frac{dy}{dx} + y^2 - \left[x^3\frac{dy}{dx} + y(3x^2)\right] = 0$$

$$2xy\frac{dy}{dx} + y^2 - x^3\frac{dy}{dx} - 3x^2y = 0$$

$$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 3x^2y - y^2$$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$$

p. 180 QQ #4

- A curve in the xy-plane is defined by $xy^2 - x^3y = 6$

b) Find an equation for the tangent line at each point on the curve with x-coordinate 1.

$xy^2 - x^3y = 6$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3 \quad y = -2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

$(1, 3)$

$m = \frac{9-9}{6-1}$

$m = 0$

$y = 3 + 0(x-1)$

$\boxed{y = 3}$

$(1, -2)$

$m = \frac{-6-4}{-4-1}$

$m = \frac{-10}{-5}$

$m = 2$

$\boxed{y = -2 + 2(x-1)}$

p. 180 QQ #4

- A curve in the xy-plane is defined by $xy^2 - x^3y = 6$

c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

$\frac{dy}{dx}$ is undefined

$dx = 0$

$2xy - x^3 = 0$ ✓

$2xy = x^3$

$y = \frac{x^3}{2x} = \frac{x^2}{2}$

$xy^2 - x^3y = 6$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$\frac{x^5}{4} - \frac{2x^5}{4} = 6$

$\frac{-x^5}{4} = 6$

$x^5 = -24$

$x = \sqrt[5]{-24}$

p. 162 #11

- Find dy/dx of the curve and the slope of the curve at the indicated point.

$$(x-1)^2+(y-1)^2=13 \qquad (3,4)$$

$$2(x-1)+2(y-1)\frac{dy}{dx}=0$$

$$2(3-1)+2(4-1)\frac{dy}{dx}=0$$

$$4+6\frac{dy}{dx}=0$$

$$6\frac{dy}{dx}=-4$$

$$\frac{dy}{dx}=-\frac{4}{6}=-\frac{2}{3}$$

p. 162 #12

- Find dy/dx of the curve and the slope of the curve at the indicated point.

$$(x+2)^2+(y+3)^2=25 \qquad (1,-7)$$

$$2(x+2)+2(y+3)\frac{dy}{dx}=0$$

$$2(1+2)+2(-7+3)\frac{dy}{dx}=0$$

$$6-8\frac{dy}{dx}=0$$

$$6=8\frac{dy}{dx}$$

$$\frac{6}{8}=\frac{dy}{dx}$$

$$\boxed{\frac{3}{4}=\frac{dy}{dx}}$$

p. 162 #13

- Find where the slope of the curve <u>is defined</u>.

everywhere but $y = \frac{x}{2}$

$$x^2 y - xy^2 = 4$$

$$x^2\frac{dy}{dx} + y(2x) - \left[x\, 2y\frac{dy}{dx} + y^2\right] = 0$$

$$x^2\frac{dy}{dx} + 2xy - 2xy\frac{dy}{dx} - y^2 = 0$$

$$x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = y^2 - 2xy$$

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

$dx = 0$ slope und

$$x^2 - 2xy = 0$$

$$x^2 = 2xy$$

$$\frac{x^2}{2x} = y$$

$$y = \frac{x}{2}$$

p. 162 #17

- Find the lines that are tangent and normal to the curve at the point (2,3)

$$x^2 + xy - y^2 = 1$$

$(2,3)$ $m = \frac{7}{4}$

$2x + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0$

$y = 3 + \frac{7}{4}(x-2)$

$4 + 2\frac{dy}{dx} + 3 - 6\frac{dy}{dx} = 0$

$y = 3 - \frac{4}{7}(x-2)$

$7 - 4\frac{dy}{dx} = 0$

$7 = 4\frac{dy}{dx}$

p. 162 #27

- Find the first and second derivative of the following

$$x^2 + y^2 = 1$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$x + y\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = -x$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$\frac{-y + x\left(\frac{-x}{y}\right)(y)}{y^2(y)}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y^3}}$$

Bonus

- Find dy/dx if $x+\sin(xy)=y$

$$1+\cos(xy)\left[x\frac{dy}{dx}+1\right]=\frac{dy}{dx}$$

$$1+x\cos(xy)\frac{dy}{dx}+y\cos(xy)=\frac{dy}{dx}$$

$$1+y\cos(xy)=\frac{dy}{dx}-x\cos(xy)\frac{dy}{dx}$$

$$1+y\cos(xy)=\frac{dy}{dx}\left(1-x\cos(xy)\right)$$

$$\boxed{\frac{dy}{dx}=\frac{1+y\cos(xy)}{1-x\cos(xy)}}$$

p. 242 2 Use L'hopitals rule to find the limit

$$\lim_{x \to 0} \frac{\sin(5x)}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{5\cos(5x)}{1} = \frac{5}{1} = 5$$

p. 242 6 Use L'hopitals rule to find the limit

$$\lim_{x \to \frac{\pi}{2}} \frac{1-\sin(x)}{1+\cos(2x)} = \frac{0}{0}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{-\cos(x)}{-2\sin(2x)} = \frac{0}{0}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{-4\cos(2x)} = \frac{1}{4}$$

p. 242 8

- Use L'hopitals rule to find the limit

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^3 - 12x + 16} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{2x-4}{3x^2-12} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{2}{6x} = \frac{2}{12} = \frac{1}{6}$$

p. 242 16 Use L'hopitals rule to find the limit

$$\lim_{x\to\infty} \frac{5x^2-3x}{7x^2+1} = \frac{5}{7}$$

p. 242 27 Use L'hopitals rule to find the limit

$$\lim_{x \to \infty} \frac{\ln(x^5)}{x} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{1}{x^5} \cdot 5x^4 = \frac{5}{x} \to 0$$

p. 242 35 Use L'hopitals rule to find the limit

$$\lim_{x \to \infty} \frac{\log_2 x}{\log_3(x+3)} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{\frac{1}{x \ln 2}}{\frac{1}{(x+3)\ln 3}} = \frac{(x+3)\ln 3}{x \ln 2} \rightarrow \frac{\ln 3}{\ln 2}$$

$$\lim_{x \to \infty} \frac{\ln 3}{\ln 2}$$