Related Rates

p. 251 11

$\frac{dV}{dt} = 100\pi ft^3/min$

$r = 5$

- A spherical balloon is inflated with helium at the rate of 100π ft^3/min.

a) How fast is the balloon's radius increasing at the instant the radius is 5 feet?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$100\pi = 4\pi(5^2)\frac{dr}{dt}$

$100\pi = 100\pi \frac{dr}{dt}$

$1 \text{ ft/min} = \frac{dr}{dt}$

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- A spherical balloon is inflated with helium at the rate of 100π ft^3/min.

b) How fast is the surface area increasing at that instant?

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dS}{dt} = 8\pi(5)\left(\frac{1}{}\right)$

$= 40\pi \text{ ft}^2/\text{min}$

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$\frac{dV}{dt} = 50 m^3/min \quad h = 5 m$

- Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. dh/dt

a) How fast is the water level falling when the water is 5 m deep?

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (7.5h)^2 h$

$V = \frac{56.25}{3}\pi h^3$

$\frac{dV}{dt} = 56.25 h^2 \frac{dh}{dt}$

$50 = 56.25(5)^2 \frac{dh}{dt}$

$50 = 1406.25\pi \frac{dh}{dt}$

$.0113 \frac{m}{min} = \frac{dh}{dt}$

$\frac{r}{h} = \frac{45}{6}$

$\frac{6r}{6} = \frac{45h}{6}$

$r = 7.5h$

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- Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

b) How fast is the radius of the water's surface changing at that same moment?

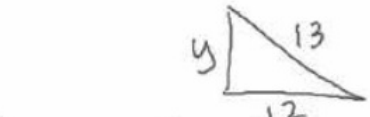
$$r = 7.5h$$

$$\frac{dr}{dt} = 7.5\frac{dh}{dt}$$

$$\frac{dr}{dt} = 7.5(.0113) = .08488 \text{ m/min}$$

p. 251 19a

$x = 12 \quad \frac{dx}{dt} = 5$

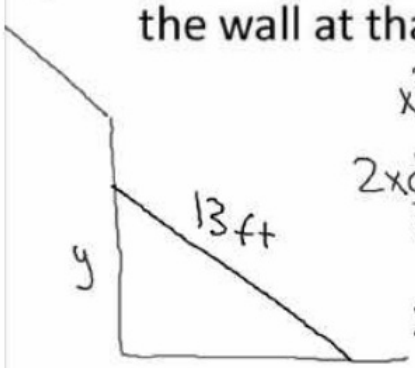


- A 13 ft ladder is leaning against a house when it's base starts to slide away. By the time the base is 12 feet from the house the base is moving at a rate of 5 ft/sec. 12 ft/min

a) How fast is the top of the ladder sliding down the wall at that moment?

$y^2 + 12^2$

$y =$



$x^2 + y^2 = 13^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(12)(5) + 2(5)\frac{dy}{dt} = 0$

$120 + 10\frac{dy}{dt} = 0$

$10\frac{dy}{dt} = -120$

$\frac{dy}{dt} = -12 \text{ ft/min}$