

Calculus BC **Words** and **Phrases** and what they should mean to you

Limits and Continuity

Word/Phrase	What does it mean to you
Find $\lim_{x \rightarrow 0} \frac{f(x) + 1}{\sin x}$ if $f(0) = -1$ and $f'(0) = 2$	Plug in 0 You get 0 over 0 Do more algebra or use L'hopitals rule Re-evaluate the limit at $x = 0$
Use the definition of the derivative	$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ or $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$
Continuous	The function has no discontinuities (no vertical asymptotes, holes, or jumps) These are usually piecewise functions. If you need to, set the two functions equal to each other and plug in the value x where the piecewise function might or might not be continuous
Continuous, but not differentiable	Cusps Corners Vertical Tangents
Given a function find the horizontal and vertical asymptotes	$\lim_{x \rightarrow \pm\infty} f(x) = \text{Horizontal asymptote}$ Set the denominator = 0 to find vertical asymptotes
Horizontal Asymptote	$\lim_{x \rightarrow \pm\infty} f(x) = \text{Horizontal asymptote}$

Differentiable

Differentiable	The function is continuous with no corners or cusps If you are given a piecewise function, the two original pieces must equal each other and the derivatives of the two pieces must equal each other
Twice Differentiable	The function has a first and second derivative.

Derivatives

Average Rate of Change

Average Rate of Change	Slope Must show difference quotient $\frac{f(b) - f(a)}{b - a}$
Find the average acceleration given the velocity as a table of values	Slope between points in the table Must show difference quotient $\frac{f(b) - f(a)}{b - a}$
Find the average rate of Change of A(t) given A(t)	Slope Must show difference quotient $\frac{f(b) - f(a)}{b - a}$

Absolute Max and Minimum from an equation

Word/Phrase	What does it mean to you
Most/Farthest/Absolute Min or Max	<p>Find any critical points by setting the first derivative equal to 0</p> <p>Plug endpoints and critical points back into the original</p> <p>Sometimes on these you may have to find the areas if given a graph or do the antiderivative if given an equation of the rate</p>
Given $f'(x)$, when does f attain a max on an interval	<p>Set $f'(x) = 0$</p> <p>This will give you your critical point</p> <p>Since it is asking for a max on an interval plug the critical point and the endpoints of your interval into the original.</p> <p>The highest value will be the max</p>

Derivative

Estimate W'	<p>An estimate probably means to use a table of values to find the derivative</p> <p>So just do slope</p> <p>If this is a calculator problem just use Math 8</p> <p>If you need to interpret this make sure you say W is increasing or decreasing at the given time and include the number with units</p>
Find the value of $A'(t)$ given A(t)	<p>Take the derivative plug in a value</p> <p>If this is a calculator problem just use Math 8</p>
Given an equation that represents the total find the rate at a certain time	Take the derivative
$\frac{d}{dx}$	Take the derivative

Tangent and Normal Lines

<p>Determining Tangent line Approximations with information about the second derivative</p>	<p>Tangent line is an under approximation if the second derivative is positive (f is concave up)</p> <p>Tangent line is an over approximation if the second derivative is negative (f is concave down)</p>
<p>Equation of the line tangent to</p>	<p>Find the point and find the slope</p> $y = y_1 + \frac{dy}{dx}(x - x_1)$
<p>Find the linear approximation, L(t)</p>	<p>Tangent Line</p> <p>Point and slope from a derivative</p> <p>Plug in a given value to the derivative</p>
<p>Horizontal Tangent</p>	<p>The slope is zero</p> <p>If given $\frac{dy}{dx}$, set $dy = 0$.</p>
<p>Normal Line</p>	<p>Perpendicular</p> <p>We need a point and the slope (which comes from the derivative, then do the opposite sign and flip it)</p> $y - y_1 = m(x - x_1)$
<p>Parallel Tangent Lines</p>	<p>Two line that have the same slope, which means they have the same derivative.</p>

Instantaneous Rate of Change

Instantaneous Rate of Change	Take the derivative
Instantaneous rate of change vs. Average rate of change vs Average Value	Instantaneous Rate of change is the derivative Average rate of change is slope Average value is $V = \frac{1}{b-a} \int_a^b f(x)dx$
Rate of Change at a point	Take the derivative

Slope of Inverse Functions

If g is inverse of f find $g'(x)$ give f	Take the slope of f and flip Inverses just switch x and y, so you just flip the value of the derivative which is the slope
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Increasing and Decreasing

Decreasing	$\frac{dy}{dx} < 0, f'(x) < 0$ May need to find critical points from first derivative and do a sign analysis. Remember to plug values into the first derivative only
Increasing	$\frac{dy}{dx} > 0, f'(x) > 0$ May need to find critical points from first derivative and do a sign analysis. Remember to plug values into the first derivative only

Mean Value Theorem

<h3>Mean Value Theorem</h3>	$f'(c) = \frac{f(b) - f(a)}{b - a}$ <p>Find where the derivative equals the slope between the endpoints of the given interval (tangent line equal to secant line)</p> <p>These can be problems from a table or problems from a graph</p> <p>The function</p>
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Related Rates

Find the rate at which the distance between 2 curves is changing with respect to time	Subtract the 2 curves to create a function and then take the derivative of that function $\frac{dx}{dt}$
Find the rate in meters per minute	Take the derivative with respect to time $\frac{dx}{dt}$
Rate of Change with respect to time	Remember to take the derivative as $\frac{dx}{dt}$
Find dy/dt given y as a function of x	$y = 2x^2$ $\frac{dy}{dt} = 4x \frac{dx}{dt}$
Increasing at a rate with respect to time	$\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}, \frac{dr}{dt}, \frac{dV}{dt}, \frac{dA}{dt}$

Local/Relative Extrema

Local/Relative Maximum	<p>Set the first derivative equal to zero to find the critical points</p> <p>Perform a sign analysis to the left and right of the critical point(s)</p> <p>Remember to show the values you get when you plug into the derivative</p> <p>First derivative values will change from negative to positive</p>
Local/Relative Minimum	<p>If a sign analysis does not work, you may need to check the concavity using the second derivative at the critical point</p> <p>$f''(x) > 0$ the curve would be concave up at the critical point making the critical point a local minimum</p> <p>$f'' < 0$ the curve would be concave down at the critical point making the critical point a local maximum</p>
Critical Points/Values	Find where $f'(x) = 0$ and $f'(x)$ is undefined

Second Derivative

Find $\frac{d^2b}{dt^2}$ given $\frac{dB}{dt}$	Take the derivative, this will be the second derivative
Concave Up	<p>$f''(x) > 0$</p> <p>May need to find possible points of infection from second derivative and do a sign analysis. Remember to plug values into the second derivative only</p>
Concave Down	<p>$f'' < 0$</p> <p>May need to find possible points of infection from second derivative and do a sign analysis. Remember to plug values into the second derivative only</p>

Inflection Points

<p>Inflection points given the first derivative function</p>	<p>Take the derivative and set the derivative equal to 0 (This derivative will be the second derivative, since you were given the first)</p> <p>Perform a sign analysis to the left and the right of the possible inflection point. Plug values into 2nd derivative.</p> <p>To be an inflection point the values of the second derivative must change</p>
<p>Inflection points given $y=$</p>	<p>Take the first 2 derivatives and set the second derivative equal to 0</p> <p>Perform a sign analysis to the left and the right of the possible inflection point. Plug values into 2nd derivative.</p> <p>To be an inflection point the values of the second derivative must change</p>
<p>Point of Inflection</p>	<p>Find the point of inflection by finding $f''(x)$ then setting $f''(x) = 0$ and checking when $f''(x)$ is undefined. Then check points left and right of the P.I.P.S. $f''(x) = 0$ must change sign</p>

Velocity

<p>Given velocity, how many times does a particle change directions</p>	<p>If you know the velocity function, set it equal to zero and do a sign analysis. When $v(t)$ signs change it gives you the number of directions changes.</p>
<p>Velocity</p>	<p>Take the derivative of the position function</p>

Acceleration

<p>Acceleration of a particle in terms of $y = f(x)$</p>	<p>Take 2 derivatives</p>
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Given a graph

Acceleration of a particle given the graph of velocity	Find the slope of the graph
Average Rate of Change of $f(x)$ given the graph of $f'(x)$	Find the area using geometry then divide by the width of the interval
Find $g(3)$ given $g(x) = \int_{-3}^x f(t)dt$ and the graph of f	Find the area under the curve from -3 to 3 Remember areas below the x-axis are negative and areas above the x-axis are positive unless you are working backwards from right to left on the graph
Find where g is concave up given $g(x) = \int_{-3}^x f(t)dt$ and the graph of f	The given graph will be the derivative, so g will be concave up when the slope of the derivative is positive
Find where g is increasing given $g(x) = \int_{-3}^x f(t)dt$ and the graph of f	The given graph will be the derivative, so g is increasing when the derivative is positive, so the given graph will be above the x-axis

<p>Given graph of $f(x)$ and</p> $g(x) = \int_1^x f(t)dt$ <p>find $g(-2)$</p>	<p>Find the area from $x = 1$ to $x = -2$</p> <p>This area will be the opposite of what is shown on the graph since you are working back from right to left</p>
<p>Given graph of $f(x)$ and</p> $g(x) = \int_1^x f(t)dt$ <p>find $g'(2)$</p>	<p>Take the derivative. In this case $g'(x) = f(x)$</p> <p>The graph they give you will be the derivative, find the y-value of the graph at $x = 2$</p>
<p>Given graph of $f(x)$ and</p> $g(x) = \int_1^x f(t)dt$ <p>find $g''(2)$</p>	<p>Take the derivative. In this case $g'(x) = f(x)$</p> <p>The graph they give you will be the derivative, so $g''(x)$ will be the slope at $x = 2$</p>
<p>Given graph of $f(x)$ and</p> $g(x) = \int_1^x f(t)dt$ <p>find $g(2)$</p>	<p>Find the area from $x = 1$ to $x = 2$</p>

<p>Given graph of $f(x)$ and</p> $g(x) = \int_1^x f(t)dt$ <p>find when $f(x)$ has a point of inflection</p>	<p>Take the derivative. In this case $g'(x) = f(x)$</p> <p>The graph they give you will be the derivative, so $g''(x)$ will be the slope of the given graph</p> <p>Look for maximums and/or minimums of the graph of the derivative</p>
<p>Given graph of $f(x)$ and</p> $g(x) = \int_1^x f(t)dt$ <p>find when $f(x)$ has an absolute max or min</p>	<p>Take the derivative. In this case $g'(x) = f(x)$</p> <p>The graph they give you will be the derivative, The critical points will be the endpoints say $[a, b]$ and where the area changes from positive to negative, say c.</p> <p>You will then need to find the areas to see which is bigger, because this is plugging endpoints and critical points back into the original</p> $g(a) = \int_a^a f(t)dt =$ $g(a) = \int^c f(t)dt =$ $g(a) = \int^b f(t)dt =$

<p>Given graph of $f(x)$ and</p> $g(x) = \int_1^x f(t) dt$ <p>find when $f(x)$ is increasing and/or decreasing</p>	<p>Take the derivative. In this case $g'(x) = f(x)$</p> <p>The graph they give you will be the derivative, so look where the area is positive for increase and where the area is negative for decrease</p>
<p>Given the graph of the derivative of f, find when f has a local minimum</p>	<p>This will occur when the graph of the derivative crosses the x-axis and the graph moves from below the x-axis to above the x-axis</p> <p>You could also look at where the area changes from negative to a positive</p>
<p>Given the graph of the derivative of f, find when f has an absolute minimum on a given interval</p>	<p>This will occur when the graph of the derivative crosses the x-axis and the graph moves from below the x-axis to above the x-axis</p> <p>This may also occur at an endpoint because we are looking for an absolute minimum</p> <p>You must not look at the areas under the graph of the derivative to see which is the smallest</p>
<p>Given the graph of the derivative of f, find when f is both concave down and increasing</p>	<p>You will be looking for a section of the derivative graph that is above the x-axis (shows f is increasing) and whose slope is negative (slope of derivative graph shows concavity)</p>
<p>Points of inflection of f given graph of the derivative</p>	<p>Points of Inflection of f will occur where the maximums and minimums of the derivative occur because the slope of the derivative changes at these values</p>
<p>Given a graph find speed</p>	<p>Speed is increasing when $a(t)$ and $v(t)$ have the same sign</p> <p>Speed is decreasing when $a(t)$ and $v(t)$ have opposite signs</p>

Given a table

<p>Estimate average value using left Riemann sums</p>	<p>From a table use $A = lw$</p> <p>The lengths will come from the y-values (use the first y-value, but not the last)</p> <p>The widths will come from the difference in x-values</p>
<p>Given a table of values find $C'(3.5)$</p>	<p>Slope</p> <p>Show the difference quotient</p>
<p>Given a table of values find out if there is a time that $C'(t) = 2$</p>	<p>Check the slopes between the given values in the table</p> <p>If there is a pair of consecutive slopes that are below 2 and above 2 then that is the interval where the slope equals 2</p>
<p>Given a table, Use a midpoint sum to find $\frac{1}{6} \int_0^6 C(t) dt$ and Explain the meaning in context</p>	<p>From a table use $A = lw$</p> <p>The lengths will come from the y-values (use the y-value in the middle of each interval)</p> <p>The widths will come from the difference in every other x-value</p> <p>Then divide the answer by 6</p> <p>The meaning should have something to do with the average</p>

<p>How many times do the data reach a certain value...</p> <p>Do the data support reaching a certain value...</p> <p>How many values...</p> <p>Explain why there must be a value of...</p>	<p>Intermediate Value Theorem</p> <p>Since the function is continuous (1 point for saying continuous) there exists so many values because the value in question is in between 2 given values</p> <p>Make sure you identify the two values that the value wanted must be between</p>
<p>Instantaneous rate of change given a table</p>	<p>Slope</p> <p>Show the difference quotient</p>
<p>Intermediate Value Theorem</p>	<p>If the function is continuous then every y-value between the y-values of the endpoints will be accounted for</p> <p>For example: On the interval [1,5] if (1, 9) and (5,20) are the endpoints we know that every y-value between 9 and 20 will exist on the graph of f(x) if it is continuous</p>
<p>Left Riemann Sum</p>	<p>From a table use $A = lw$</p> <p>The lengths will come from the y-values (use the first y-value, but not the last)</p> <p>The widths will come from the difference in x-values</p> <p>Left Riemann Sums are under approximations if f(x) is increasing</p> <p>Left Riemann Sums are over approximations if f(x) is decreasing</p>

<p>Midpoint Approximation</p>	<p>From a table use $A = lw$</p> <p>The lengths will come from the y-values (use the y-value in the middle of each interval)</p> <p>The widths will come from the difference in every other x-value</p>
<p>Right Riemann Sum</p>	<p>From a table use $A = lw$</p> <p>The lengths will come from the y-values (use the last y-value, but not the first)</p> <p>The widths will come from the difference in x-values</p> <p>Right Riemann Sums are over approximations if $f(x)$ is increasing</p> <p>Right Riemann Sums are under approximations if $f(x)$ is decreasing</p>
<p>Trapezoidal Sum given a table</p>	<p>$\frac{1}{2}h(b_1 + b_2)$</p> <p>h comes from change in x – values</p> <p>bases come from y-values</p>
<p>Evaluate $\int W'(t)$</p>	<p>$\int_a^b W'(t) = [W(t)]_a^b = W(b) - W(a)$</p>

Slope Fields

Slope Field	Plug values into the derivative and sketch the slopes
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Integrals

Average Value

Average Value	$A = \frac{1}{b-a} \int_a^b f(x) dx$ <p>Area divided by the width of the interval</p>
Find $\frac{1}{20} \int W'(t)$	<p>Average Value</p> $\frac{1}{20} \int_a^b W'(t) = \frac{[W(t)]_a^b}{20} = \frac{W(b) - W(a)}{20}$
Write an expression for the average value	$\frac{1}{b-a} \int_a^b f(x) dx$
Average value given $f(x)$	<p>Area divided by width</p> $\frac{1}{b-a} \int_a^b f(x) dx$
Find average rate given rate	<p>Average Value $A = \frac{1}{b-a} \int_a^b f(x) dx$</p>

Differential Equations

Find the particular solution given $\frac{dy}{dx}$ and an initial condition	Integrate then find C Don't forget +C Solve for y
Find the particular solution of the differential equation	Integrate the given derivative Don't forget +C Solve for C
Find the particular solution to the differential equation with initial condition	Integrate $\frac{dy}{dx}$ Don't forget C Find C Solve for y
Use separation of variables to find the particular solution to the differential equation	Take the integral/antiderivative Don't forget +C Find C Solve for y

Eulers Method

<p>Use Eulers Method</p> <p>$f(0) = -1$</p> <p>Two steps to find</p> <p>$f(.5)$</p>	<p>Previous $y + (\text{change in } x)(\text{previous slope})$</p> <p>Consecutive Tangent Lines $y = y_1 + dy/dx(x-x_1)$, where x_1, y_1 is the previous coordinate given and dy/dx is the derivative given</p> <p>Can be organized using the table below.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> <th>$\frac{dy}{dx}$</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>-1</td> <td></td> </tr> <tr> <td>.5</td> <td></td> <td></td> </tr> <tr> <td>1</td> <td></td> <td></td> </tr> </tbody> </table>	x	y	$\frac{dy}{dx}$	0	-1		.5			1		
x	y	$\frac{dy}{dx}$											
0	-1												
.5													
1													

First Fundamental Theorem of Calculus

Word/Phrase	What does it mean to you
<p>Find f' given $f(x) = \int_4^{2x} f(t)$</p>	<p>$f' = 2f(2x)$</p> <p>Substitute in the upper limit and multiply the function by the value of the upper limits derivative</p>
<p>Derivative given an integral</p>	<p>$\frac{d}{dx} \int_0^{x^3} \sin(t) dt = \sin(x^3)(3x^2)$</p> <p>Cancel the integral and then multiply by the derivative of the top limit</p>
<p>Find</p> <p>$g'(x)$ given $g(x) = \int_0^{x^2} f(t) dt$</p>	<p>Substitute in the upper limit and multiply the function by the value of the upper limits derivative</p> <p>$g'(x) = 2xf'(x^2)$</p>

Given a rate

Given a rate entering and a rate leaving, find the total amount	Integrate both rates and then subtract them
Given a rate find if the rate is increasing or decreasing	Take the derivative of the rate Find the critical points (set = 0) Plug in values to the left and right of the critical point into the derivative Any positive values of the derivative will give you increasing Any negative values of the derivative will give you decreasing
Given a rate find the maximum	Integrate the rate on the given interval Don't forget to add back the original amount With the greatest total you may have to integrate the rate from the starting point to the critical point (Set the rate = 0) and from the critical point to the endpoint. (this checks your endpoints)
Given a rate, find the amount is greatest	Integrate the rate on the given interval Don't forget to add back the original amount With the greatest total you may have to integrate the rate from the starting point to the critical point (Set the rate = 0) and from the critical point to the endpoint. (this checks your endpoints)

<p>Given the rate and an initial amount find the total</p>	<p>Integrate the rate and then add back the initial value</p> <p>Use Math 9 if it is a calculator problem</p>
<p>Given the rate $g(t)$ find $g'(5)$ and interpret the meaning in context</p>	<p>Plug in $t = 5$ into the given rate and then interpret the meaning</p>
<p>The <u>greatest</u> total given the rate</p>	<p>Integrate the rate on the given interval</p> <p>Don't forget to add back the original amount</p> <p>With the greatest total you may have to integrate the rate from the starting point to the critical point (Set the rate = 0)</p>
<p>Find the total given a rate</p>	<p>Integrate the rate</p> <p>Add back the initial amount</p> <p>These are usually calculator problems so use Math 9, but if they are not calculator just take the antiderivative</p>

Logistic Growth Functions

<p>Logistical Differential</p> <p>Logistical Growth</p>	<p>$\frac{dP}{dt} = kP(M - P)$ is the logistical growth differential equation (usually have to factor to get in this form)</p> <p>$P = \frac{M}{1 + Ae^{-mkt}}$ is the solution to the differential equation (rarely if ever have to find this)</p> <p>M-Max Capacity = $\lim_{t \rightarrow \infty} \frac{dp}{dt}$</p> <p>Growing the fastest at M/2</p>
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Position/Velocity using anti-derivative

<p>Find position given v(t) and t =4</p>	<p>Integrate v(t) and add back the initial value at t = 4</p> <p>This can be done by finding C after you integrate. Remember to include C immediately after you take the anti-derivative</p>
<p>Given a(t) and velocity at t = 0 find v(t)</p>	<p>Integrate a(t)</p> <p>Don't forget +C</p> <p>Find C</p>
<p>Position Given Velocity</p>	<p>Integrate the velocity</p> <p>Don't forget +C</p> <p>Plug in the initial value to find C then solve for y</p>
<p>Position given velocity and initial value</p>	<p>Integrate the velocity and don't forget +C</p> <p>Solve for C</p>
<p>Write an expression for the position given v(t) and the starting point</p>	<p>$s(t) = \text{starting point} + \int_{\text{starting point of } t}^{\text{ending value of } t} f(t)dt$</p>

Area

Area with respect to the x-axis	Everything in the integral must be in terms of x (solve equation for y) $A = \int (\text{Top Curve}) - (\text{Bottom Curve})$
Area with respect to the y-axis	Everything in the integral must be in terms of y (solve equation for x) $A = \int (\text{Right Curve}) - (\text{Left Curve})$
Find the area of R given 2 curves	$\int (\text{Top} - \text{Bottom}) dx \quad \text{All } x$ <p><i>or</i></p> $\int (\text{Right} - \text{Left}) dy \quad \text{All } y$

Volume

<p>Find the volume of the solid rotated around a horizontal line that is below the region given</p>	$V = \pi \int (\text{Outer radius})^2 - (\text{Inner Radius})^2 dx \quad \text{All } x$ $V = \pi \int (\text{Top Curve- Line})^2 - (\text{Bottom Curve- Line})^2 dx \quad \text{All } x$
<p>Volume rotated around the x-axis</p>	$\pi \int_{x_1}^{x_2} (R)^2 dx$ <p>Remember everything in the integral must be in terms of x (Solve the equation for y)</p> <p>Your radius is always your curve</p> $\pi \int_{x_1}^{x_2} (a - R)^2 dx$ <p>If you rotate about a line, that line must be in your equation</p> <p>Your radius is always your curve</p> <p>If your shaded region does not touch the line you are rotating around, you will have two function</p> $\pi \int_{x_1}^{x_2} (R_1)^2 - (R_2)^2 dx$
<p>Volume rotated around the y-axis</p>	$\pi \int_{y_1}^{y_2} (R)^2 dx$ <p>Remember everything in the integral must be in terms of x (solve the equation for x) and your radius is always your curve</p> $\pi \int_{y_1}^{y_2} (a - R)^2 dx$ <p>If you rotate about a line, that line must be in your equation</p> <p>Your radius is always your curve</p> <p>If your shaded region does not touch the line you are rotating around, you will have two function</p> $\pi \int_{y_1}^{y_2} (R_1)^2 - (R_2)^2 dx$

Volume using right isosceles triangles	<p>The distance between the curves you are given will be the base and height of your triangle.</p> $V = \int \frac{1}{2}bh$
Volume using semi-circle cross sections	<p>The distance between the curves you are given will be the diameter of your circle</p> $V = \frac{\pi}{2} \int r^2$
Volume using square cross sections	<p>The distance between the curves you are given will be the base and height of your square.</p> <p>You will need to draw an equilateral triangle to find the height</p> $V = \int bh$
Volume of a cross section using equilateral triangles	<p>The distance between the curves you are given will be the base of your triangle.</p> <p>You will need to draw an equilateral triangle to find the height using Pythagorean theorem</p> $V = \int \frac{1}{2}bh$

Partial Fractions

Integration by Partial Fractions	Factor the denominator Solve for A and B Integrate using natural log (Ln)
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Tabular Integration

Tabular Integration	<p>These are integrals of products where the first function is not the derivative of the second</p> <p>Derivative of the left side of the table</p> <p>Ant-derivative of the right side of the table</p> <p>Multiply the answers diagonally</p> <p>Every other diagonal switches sign</p> <p>If the derivative of the left side does not go to 0, stop after one row of the table.</p> <ul style="list-style-type: none">- Multiply diagonally and then take the integral of the product of the last row- Don't forget plus C
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Improper Integrals

Improper Integral	$\lim_{b \rightarrow \infty} \int_2^b f(x) dx$ <p>These are usually limits involving infinity</p> <p>These will either converge to a value or diverge</p>
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Parametric/Vectors

Word/Phrase	What does it mean to you
Acceleration Vector	$\left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$ <p>Use these symbols and keep them separate</p> <p>You must take the derivative of x(t) and y(t) twice</p>
Farthest right given parametric derivatives	<p>Use $\frac{dx}{dt}$</p> <p>Set $\frac{dx}{dt}=0$ then plug the critical point from this and the endpoints back into the original function (you will have to integrate $\frac{dx}{dt}$ to get back to your original function in this case)</p>
Find the x-coordinate given a parametric derivative and an initial value	<p>Integrate $\frac{dx}{dt}$ and add back the initial x-value</p> $5 + \int \frac{dx}{dt}$
Given $\frac{dx}{dt}$ and position at $t = 2$ find the x-coordinate of particles position	<p>Integrate $\frac{dx}{dt}$ and add back the initial x-value</p> $5 + \int_0^2 \frac{dx}{dt}$

<p>Given $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>find acceleration</p>	$a(t) = \left\langle \left(\frac{dx}{dt} \right), \left(\frac{dy}{dt} \right) \right\rangle$
<p>Given $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>find distance traveled</p>	$\text{Total Distance} = \int \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}$
<p>Given $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>find speed</p>	$\text{Speed} = \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}$
<p>Given $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>find speed</p>	$\sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}$
<p>Given $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>find the tangent line</p>	<p>Point and slope</p> <p>Find the point by plugging the t value back into your original equations</p> <p>Find your slope by taking the derivative of y and putting it over the derivative of x</p> <p>Plug in your t value</p> $y - y_1 = \frac{dy}{dx} (x - x_1)$

<p>Given $\frac{dx}{dt}$ and $\frac{dy}{dt}$ find when the particle is farthest right</p>	<p>Use $\frac{dx}{dt}$</p> <p>Set $\frac{dx}{dt} = 0$ then plug the critical point from this and the endpoints back into the original function (you will have to integrate $\frac{dx}{dt}$ to get back to your original function in this case)</p>
<p>Given $\frac{dx}{dt}$ and $\frac{dy}{dt}$ horizontal movement left or right</p>	<p>Just plug in the given value of t</p> <p>$\frac{dx}{dt} > 0$ right $\frac{dx}{dt} < 0$ left</p>
<p>Tangent line in parametric</p>	<p>Point and slope</p> <p>Find the point by plugging the t value back into your original equations</p> <p>Find your slope by taking the derivative of y and putting it over the derivative of x</p> <p>Plug in your t value</p> $y - y_1 = \frac{dy}{dx}(x - x_1)$
<p>Slope given parametric equations</p>	<p>Slope is always $\frac{dy}{dx}$</p> <p>Take the derivative of y with respect to t</p> <p>Take the derivative of x with respect to t</p> <p>Put dy over dx</p>
<p>When is a particle given in parametric at rest</p>	<p>Set $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$</p> <p>Find the time they have in common</p>

Arc Length/Perimeter

Find the perimeter of the region	Arc Length $L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$
Length or total distance traveled given a set of parametric equations	Arc Length $L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

Polar

Word/Phrase	What does it mean to you
Find $\frac{dx}{d\theta}$ given a polar equation r	Take the derivative of $x = r \cos \theta$
Find $\frac{dr}{dt}$ given a polar equation r and $\frac{d\theta}{dt}$	Take the derivative of r with respect to time Interpret $\frac{dr}{dt}$ as the rate at which the radius is changing with respect to time Interpret $\frac{d\theta}{dt}$ as how close or how far away from the origin the particle is at a given time
Find the area given polar (calculator okay)	$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$
Given a polar equation (r=) find the position vector	$x = r \cos \theta \quad y = r \sin \theta$
Given a polar equation (r=) find the velocity vector	$x = r \cos \theta \quad y = r \sin \theta$ Then take the derivative and use the right notation $\left\langle \frac{dx}{d\theta}, \frac{dy}{d\theta} \right\rangle$

<p>Given a polar equation ($r=$) find when the x-coordinate is -1</p>	<p>$x = r \cos \theta$</p> <p>Let $x = -1$ and solve for θ</p>
<p>Given a polar function find the x-coordinate or y-coordinate</p>	<p>$x = r \cos \theta \quad y = r \sin \theta$</p>
<p>Area of a polar equation</p>	<p>$A = \frac{1}{2} \int r^2 d\theta =$</p> <p>Remember the limits should be in terms of $\theta =$</p> <p>Use your calculator if you can</p> <p>If not use some algebra to solve a trig equation</p>
<p>The rate distance is changing with respect to time between 2 polar curves</p>	<p>Create a function by subtracting the outer curve minus the inner curve</p> <p>Then take the derivative with respect to time</p>
<p>Slope of a line tangent to a curve in polar ($r =$)</p>	<p>$x = r \cos \theta, y = r \sin \theta$</p> <p>Find dy/dx by using the product rule on each</p>

Series

Word/Phrase	What does it mean to you
$ f(x) - P(x) \leq \frac{1}{100}$	$\frac{(\text{max of next derivative})(\text{distance from center})^n}{n!}$ $ P_{3(x)} - f(x) \leq \frac{(f^4(x))(x-a)^4}{4!}$ <p>Next term if the series alternates</p>
$ x - 1 < R$	<p>This symbolizes the radius of convergence</p> <p>This would mean your polynomial is centered at $x = 1$</p>
<p>Estimate differs from actual value (series)</p>	$\frac{(\text{max of next derivative})(\text{distance from center})^n}{n!}$ $ P_{3(x)} - f(x) \leq \frac{(f^4(x))(x-a)^4}{4!}$ <p>Next term if the series alternates</p>
<p>Find $P_3(x)$ given info about the first 3 derivatives</p>	$P_3(x) = f(a) + \frac{f'(a)(x-a)^1}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!}$
<p>Find the 2nd degree polynomial</p>	<p>Memorize your Maclaurin Series centered at $x = 0$, substitute into these as necessary</p> <p>If the series is not a famous Maclaurin series centered at $x = 0$, then you will have to take derivatives, plug the center into the original function and the derivatives and build your polynomial by putting the value at the center over the factorial</p> $P_2(x) = f(a) + \frac{f'(a)(x-a)^1}{1!} + \frac{f''(a)(x-a)^2}{2!}$

<p>Find the 3rd degree Taylor polynomial</p>	<p>Memorize your Maclaurin Series centered at $x = 0$, substitute into these as necessary</p> <p>If the series is not a famous Maclaurin series centered at $x = 0$, then you will have to take derivatives, plug the center into the original function and the derivatives and build your polynomial by putting the value at the center over the factorial</p> $P_3(x) = f(a) + \frac{f'(a)(x-a)^1}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!}$
<p>Find the first 4 non-zero terms and general term</p>	<p>Memorize your Maclaurin Series centered at $x = 0$, substitute into these as necessary</p> <p>If the series is not a famous Maclaurin series centered at $x = 0$, then you will have to take derivatives, plug the center into the original function and the derivatives and build your polynomial by putting the value at the center over the factorial</p> $P_2(x) = f(a) + \frac{f'(a)(x-a)^1}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots + \frac{f^{(n)}(a)(x-a)^n}{n!}$
<p>Find the first three nonzero terms and the general term of the Taylor Series for given a rule for f</p>	<p>Memorize your Maclaurin Series centered at $x = 0$, substitute into these as necessary</p> <p>If the series is not a famous Maclaurin series centered at $x = 0$, then you will have to take derivatives, plug the center into the original function and the derivatives and build your polynomial by putting the value at the center over the factorial</p> <p>The first 3 means 3 terms, not 3 derivatives</p>
<p>Find the general Taylor Series centered at $x = 0$</p>	<p>Memorize your Maclaurin Series centered at $x = 0$, substitute into these as necessary</p> <p>If the series is not a famous Maclaurin series centered at $x = 0$, then you will have to take derivatives, plug the center into the original function and the derivatives and build your polynomial by putting the value at the center over the factorial</p> $P_n(x) = f(0) + \frac{f'(0)(x)^1}{1!} + \frac{f''(0)(x)^2}{2!} + \frac{f'''(0)(x)^3}{3!} + \dots + \frac{f^{(n)}(0)(x)^n}{n!}$

<p>Find the general Taylor Series centered at $x = 2$</p>	<p>Memorize your Maclaurin Series centered at $x = 0$, substitute into these as necessary</p> <p>If the series is not a famous Maclaurin series centered at $x = 0$, then you will have to take derivatives, plug the center into the original function and the derivatives and build your polynomial by putting the value at the center over the factorial</p> $P_2(x) = f(2) + \frac{f'(2)(x-2)^1}{1!} + \frac{f''(2)(x-2)^2}{2!} + \dots + \frac{f^{(n)}(2)(x-2)^n}{n!}$
<p>Find the radius of convergence</p>	<p>Ratio Test</p>
<p>Find the second degree Taylor Polynomial</p>	<p>Memorize your Maclaurin Series centered at $x = 0$, substitute into these as necessary</p> <p>If the series is not a famous Maclaurin series centered at $x = 0$, then you will have to take derivatives, plug the center into the original function and the derivatives and build your polynomial by putting the value at the center over the factorial</p> $P_2(x) = f(a) + \frac{f'(a)(x-a)^1}{1!} + \frac{f''(a)(x-a)^2}{2!}$
<p>For an alternating series show that the approximation differs by less than $1/200$</p>	$\frac{(\text{max of next derivative})(\text{distance from center})^n}{n!}$ $ P_{3(x)} - f(x) \leq \frac{(f^{(4)}(x))(x-a)^4}{4!}$ <p>Next term if the series alternates</p>

<p>f' is a geometric series, find the function for f' to which the series converges to</p>	$f(x) = \frac{a}{1-r}$ <p>a – first term r – common ratio (what you are multiplying by each time)</p>
<p>f' is a geometric series, use the function to determine $x-1 < R$</p>	<p>Find the radius of convergence of the geometric series</p>
<p>Find the function to which the series converges</p>	<p>This is probably referring to a geometric series, so find $f(x) = \frac{a}{1-r}$</p> <p>a – first term r – common ratio</p> <p>If the function is not geometric it would be referring to one of your special Maclaurin series</p>
<p>Use the ratio test</p>	<p>Find the limit as you approach infinity of the next term (plug in $n+1$) times the reciprocal of the original series</p> <p>You will be finding an interval of convergence so set your result between -1 and 1</p> <p>Remember any limit < 1 converges</p> <p>Any limit > 1 diverges</p> $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ $\lim_{n \rightarrow \infty} \left \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right $

$f^{30}(3)$	<p>Find the 30th derivative at $x = 3$</p> <p>This will come from the x^{30} term because the derivative always matches the power in a Taylor polynomial</p> $\frac{f^{30}(3)(x-3)^{30}}{30!}$
<p>All values of Convergence</p>	<p>Use the ratio test (unless you can tell it is geometric) then check your endpoints by plugging back into the original series</p>
<p>Find a Taylor polynomial not knowing the function</p>	<p>There must be some information about the values of the function, first derivative, second derivative and so on.</p> <p>Don't forget $(x - \text{center})$ and your factorials</p> $\frac{(f^0(x))(x-a)^0}{0!} + \frac{(f'(x))(x-a)^1}{1!} + \frac{(f''(x))(x-a)^2}{2!} + \dots$
<p>Find the 3rd degree polynomial about $x = 3$</p>	<p>You will need 3 derivatives</p> <p>You will need to plug the center, 3, into the original function and the derivatives</p> <p>Put these values over the factorials</p> $P_3(x) = f(3) + \frac{f'(3)(x-3)^1}{1!} + \frac{f''(3)(x-3)^2}{2!} + \frac{f'''(3)(x-3)^3}{3!}$
<p>Interval of Convergence</p>	<p>Use the ratio test (unless you can tell it is geometric) then check your endpoints by plugging back into the original series</p>
<p>Lagrange Error Bound</p>	$\frac{(\text{max of next derivative})(\text{distance from center})^n}{n!}$ $\frac{(f^4(x))(x-a)^4}{4!}$

<p>Maclaurin Series</p>	$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $e^x = 1 + x + \frac{x^2}{2!} - \dots + \frac{x^n}{(n)!} + \dots \sum_{n=0}^{\infty} \frac{x^n}{(n)!}$
<p>Power series expansion of $\frac{1}{1-x}$</p>	<p>This is a geometric with a = 1 and r = x</p>
<p>Radius of Convergence</p>	<p>Ratio Test or Geometric</p> <p>The radius is the distance from the center of your interval of convergence to the endpoints</p>
<p>Sum of a Series</p>	<p>$S = \frac{a}{1-r}$ a – first term r – common ratio</p> <p>If it is not geometric it must be a pattern from one of the special MaClaurin Series</p>
<p>Value of $\sum_{n=1}^{\infty} a_n$</p>	<p>This means find the sum</p> <p>$S = \frac{a}{1-r}$ if geometric a- first term r – common ratio</p> <p>If it is not geometric it must be a pattern from one of the special MaClaurin Series</p>
<p>Value of f(2) given $\sum_{n=1}^{\infty} a_n$</p>	<p>This means find the sum</p> <p>$S = \frac{a}{1-r}$ if geometric a- first term r – common ratio</p> <p>If it is not geometric it must be a pattern from one of the special MaClaurin Series</p>

<p>What value does the series (given the terms) converge to</p>	<p>This can be geometric or it can converge to one of your other special Maclaurin Series</p> <p>For example:</p> $3 - \frac{3^3}{3!} + \frac{3^5}{5!} - \frac{3^7}{7!} + \dots$ <p>converges to $\sin(3)$ because the series fits the pattern of $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ where $x = 3$</p>
<p>Express the series as a rational function</p> <p>Find the function</p>	<p>This means find the sum</p> $S = \frac{a}{1-r}$ <p>a – first term r – common ration</p>

More things to be able to explain the meaning of.

$\int_a^b v(t) dt$ where $v(t)$ is in meters/second	Integrating meters/sec takes you back to the position in meters
$\int_a^b v(t) dt$ where $v(t)$ is in meters/second	Total Distance in meters
$\int_a^b v'(t) dt$ where $v(t)$ is in meters/second	Velocity in meters per second
$\frac{1}{b-a} \int_a^b x(t) dt$ where $x(t)$ is the number of people at the zoo	Average number of people at the zoo during the given time interval
$\frac{1}{b-a} \int_a^b v(t) dt$ where $v(t)$ is the number of people entering the zoo every minute	The average number of people entering the zoo on the given interval
$\frac{1}{b-a} \int_a^b v'(t) dt$ where $v(t)$ is the number of people entering the zoo every minute	The average rate that people are entering the zoo on the given interval

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