

PRE-CALCULUS: by Finney, Demana, Watts and Kennedy
Solving Trigonometric Equations

What you'll Learn About

Solve each trigonometric equation for θ on the interval $[0, 2\pi]$. Then give a formula for all possible angles that could be a solution of the equation.

A) $\sin \theta = \frac{\sqrt{2}}{2}$
 $\theta = 45^\circ, 135^\circ$

B) $\cos \theta = \frac{-1}{2}$
 $\theta = 120^\circ, 240^\circ$

$$\frac{\pi}{4} \pm 2\pi k$$

$$\frac{3\pi}{4} \pm 2\pi k$$

$$45 \pm 360k$$

$$135 \pm 360k$$

$$120 \pm 360k$$

$$240 \pm 360k$$

$$\frac{2\pi}{3} \pm 2\pi k$$

$$\frac{4\pi}{3} \pm 2\pi k$$



C) $\sin \theta = 1$
 $90^\circ \pm 360k$

$$\frac{\pi}{2} \pm 2\pi k$$

D) $\cos \theta = 0$
 $90^\circ, 270^\circ$

$$90^\circ \pm 360k$$

$$270 \pm 360k$$

$$\frac{\pi}{2} \pm 2\pi k$$

$$\frac{3\pi}{2} \pm 2\pi k$$

E) $\tan \theta = \sqrt{3}$
 $\theta = 60^\circ, 240^\circ$

$$60 \pm 180k \quad 240 \pm 180k$$

$$\frac{\pi}{3} \pm \pi k$$

$$\frac{4\pi}{3} \pm \pi k$$

F) $\tan \theta = -1$

$$\theta = 135^\circ, 315^\circ$$

$$135 \pm 180k$$

$$\frac{3\pi}{4} \pm \pi k$$

$$315 \pm 180k$$

$$\frac{7\pi}{4} \pm \pi k$$

$$\cos \theta = \frac{1}{2}$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \theta$$

$$\boxed{150^\circ, 330^\circ}$$

Solve each trigonometric equation for θ on the interval $[0, 2\pi]$.

A) $\cos 2\theta = \frac{1}{2}$

$$\cos^{-1}\left(\frac{1}{2}\right) = 2\theta$$

$$2\theta = \frac{60^\circ \pm 360^\circ k}{2}$$

$$\theta = 30 \pm 180^\circ k$$

$$\boxed{30, 210^\circ}$$

$$2\theta = \frac{300^\circ \pm 360^\circ k}{2}$$

$$150 \pm 180^\circ k$$

C) $\cos \frac{\theta}{3} = \frac{\sqrt{3}}{2}$

$$\frac{\theta}{3} = 30^\circ \pm 360^\circ k$$

$$\theta = 90^\circ \pm 1080^\circ k$$

$$\theta = \boxed{90^\circ}$$

$$\frac{\theta}{3} = 330^\circ \pm 360^\circ k$$

$$\theta = 990^\circ \pm 1080^\circ k$$

E) $\sin \theta = .4$

B) $\sin 3\theta = \frac{1}{2}$

$$\frac{3\theta}{3} = \frac{30^\circ \pm 360^\circ k}{3}$$

$$\theta = 10 \pm 120^\circ k$$

$$10^\circ, 130^\circ, 250^\circ$$

$$3\theta = 150 \pm 360^\circ k$$

$$\theta = 50^\circ \pm 120^\circ k$$

$$50^\circ, 170^\circ, 290^\circ$$

D) $\tan\left(\frac{\theta}{2} + \frac{\pi}{3}\right) = 1$

$$\frac{\theta}{2} + 60 = 45$$

$$-60 \quad -60$$

$$\frac{\theta}{2} = -15$$

$$\theta = -30^\circ$$

$$\frac{\theta}{2} + 60 = 225$$

$$\left(\frac{\theta}{2}\right) = (165)$$

$$\theta = 330^\circ$$

F) $\cos \theta = -.2$

(0, 360)