Pre-Calculus Notes
Geometric Sequence and Series

Geometric Sequence – A sequence of numbers where each term after the first is found by multiplying the previous one by the same number. That number is called the common ratio.

EX: 4, 20, 100, 500, 2500, ... r = 5

EX: 80, 40, 20, 10, 5, 5/2 ... r = ½

EX: 3, -9, 27, -81, 243, ... r = -3

Determine if the sequence is geometric. If it is find the common ratio.

1. -1, 6, -36, 216, ...

2. -1, 1, 4, 8, ...

3. 4, 16, 36, 64, ...

4. -3, -15, -75, -375, ...

5. -6, -9, $-\frac{27}{2}$, $-\frac{81}{4}$, ... Yes $r = \frac{3}{2}$

6. 1, -5, 25, -125, ...

Write the first 5 terms of the geometric sequence.

EX $a_1 = 6,\ r = 2$

$$a_1 = 6, a_2 = 12, a_3 = 24, a_3 = 48, a_4 = 96, a_5 = 192$$

7. $a_1 = -2,\ r = 5$

-2, -10, -50, -250, -1250

8. $a_1 = 3, r = -2$

9. $a_1 = \frac{1}{4}, r = \frac{1}{2}$

10. $a_1 = 90, r = -\frac{1}{3}$

Recursive Rule – Using the previous term to find the current term.

$$a_1 = first\ term \qquad a_{k+1} = r \cdot a_k$$

Use the recursive e rule to write the first 5 terms.

EX: $a_1 = 2, a_{k+1} = 3 \cdot a_k$

$a_1 =$

$a_{k+1} = a_k \cdot r$

$\boldsymbol{a_1 = 2}$
$a_{1+1} = 3 \cdot a_1, \quad a_2 = 3 \cdot 2 = 6$
$\boldsymbol{a_2 = 6}$
$a_{2+1} = 3 \cdot a_2, \quad a_3 = 3 \cdot 6 = 18$
$\boldsymbol{a_3 = 18}$
$a_{3+1} = 3 \cdot a_3 \quad a_4 = 3 \cdot 18 = 54$
$\boldsymbol{a_4 = 54}$
$a_{4+1} = 3 \cdot a_4, \quad a_5 = 3 \cdot 54 = 162$
$\boldsymbol{a_5 = 162}$

11. $a_1 = 36,\ a_{k+1} = \frac{1}{2} \cdot a_k$

$a_1 = 36$

$a_2 = 36 \cdot \frac{1}{2} = 18$

$a_3 = 18 \cdot \frac{1}{2} = 9$

$a_4 = 9 \cdot \frac{1}{2} = \frac{9}{2}$

$a_5 = \frac{9}{2} \cdot \frac{1}{2} = \frac{9}{4}$

12. $a_1 = \frac{2}{3}, a_{k+1} = 3 \cdot a_k$

13. $a_1 = 4, a_{k+1} = -2 \cdot a_k$

14. $a_1 = -100, a_{k+1} = \frac{2}{5} \cdot a_k$

Explicit Rule – Designates the nth term of a sequence, as an expression of n (where n = the terms location).

$a_n = a_1(r)^{n-1}$ or $a_n = a_0(r)^n$ *if the first term is* a_0 *not* a_1

Write the explicit rule of the sequence.

EX $a_1 = 5,\ r = -3$

$a_n = 5(-3)^{n-1}$

$a_o = 14,\ r = \frac{1}{2}$

$a_n = 14\left(\frac{1}{2}\right)^n$

15. $a_1 = 8,\ r = 3$

$a_n = a_1(r)^{n-1}$

$a_n = 8(3)^{n-1}$

16. $a_1 = -3, r = \frac{5}{3}$

$a_n = -3\left(\frac{5}{3}\right)^{n-1}$

17. $a_0 = \frac{7}{8}, r = 5$

$a_n = a_0 r^n$

$= \frac{7}{8}(5)^n$

18. $a_0 = \frac{1}{2}, r = -\frac{3}{4}$

$a_n = \frac{1}{2}\left(-\frac{3}{4}\right)^n$

Write the explicit rule of each geometric sequence. Find the first terms of the sequence, and then find the 9th term.

19. $a_1 = 64, r = \frac{1}{2}$

$a_n = 64\left(\frac{1}{2}\right)^{n-1}$

$a_1 = 64,\ a_2 = 32,\ a_3 = 16,\ a_4 = 8,\ a_5 = 4$

$a_9 = 64\left(\frac{1}{2}\right)^{9-1}$

$= \frac{1}{4}$

20. $a_1 = 3, r = -2$

$a_n = 3(-2)^{n-1}$

$a_1 = 3,\ a_2 = -6,\ a_3 = 12,\ a_4 = -24$

$a_5 = 48$

$a_9 = 3(-2)^{9-1}$

$= 768$

21. $a_1 = -\frac{1}{4}, r = 2$

22. $a_1 = -2, r = -4$

2.048×10^{-4}

23. $a_0 = 400, r = \frac{1}{5}$

$a_0 = 400$

a_1

a_2

a_3

a_4

24. $a_o = 15, r = -2$