

PRE-CALCULUS: by Finney, Demana, Watts and Kennedy
Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.1: Exponential and Logistic Functions

Exponential Function
A function that can be
rewritten in the form
 $y = a \cdot b^x$, where a is
non-zero, b is positive,
and $b \neq 1$.
a: initial value at $x = 0$
b: base

Which of the following are exponential functions?
For those that are exponential functions, state the initial value and the base. For those that are not, explain.

A) $f(x) = 3^x$

B) $g(x) = 6x^{-4}$

C) $h(x) = -2 \cdot 1.5^x$

D) $h(x) = 7 \cdot -2^x$

E) $f(x) = 5 \cdot 6^\pi$

Compute the exact value of the function without using a calculator

A) $2 \cdot 4^x$ when $x = 0$

B) $2 \cdot 4^x$ when $x = -3$

C) $-2 \cdot 4^x$ when $x = 1/2$

D) $3 \cdot 8^x$ when $x = -2/3$

Determine a formula for the exponential function $g(x)$ and $h(x)$ whose values are given in the table

x	g(x)		x	h(x)
-2	4/9		-2	128
-1	4/3		-1	32
0	4		0	8
1	12		1	2
2	36		2	1/2

Given 2 points on the graph of an exponential function, find the formula

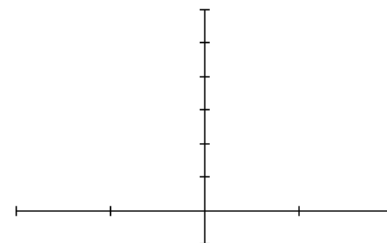
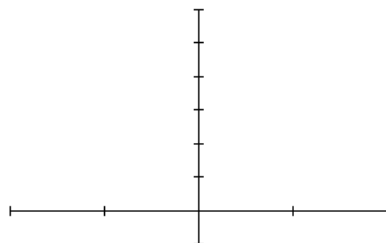
A) (0, 2) (2, 18)

B) (0.3) $\left(3, \frac{3}{e}\right)$

Sketch a graph of the following functions in the same viewing window $[-2, 2]$ $[-1, 6]$

$$y = 2^x \quad y = 3^x \quad y = 4^x \quad y = 5^x$$

$$y = \left(\frac{1}{2}\right)^x \quad y = \left(\frac{1}{3}\right)^x \quad y = \left(\frac{1}{4}\right)^x \quad y = \left(\frac{1}{5}\right)^x$$



1) Determine the domain and range

1) Determine the domain and range

2) Is the function even, odd or neither

2) Is the function even, odd or neither

3) Intervals of Increase or Decrease

3) Intervals of Increase or Decrease

4) Find any extrema.

4) Find any extrema.

5) Determine the end behavior

5) Determine the end behavior

6) Find any asymptotes

6) Find any asymptotes

7) Intervals of Concavity

7) Intervals of Concavity

Describe how to transform the graph of $f(x) = 2^x$ into the graph of g

a) $g(x) = 2^{x-1}$

b) $g(x) = 2^{-x}$

c) $g(x) = 3 \cdot 2^x$

d) $g(x) = 2^{3-x}$

Describe how to transform the graph of $f(x) = e^x$ into the graph of g

a) $g(x) = e^{4x}$

b) $g(x) = e^{-4x}$

c) $g(x) = 3 \cdot e^x + 1$

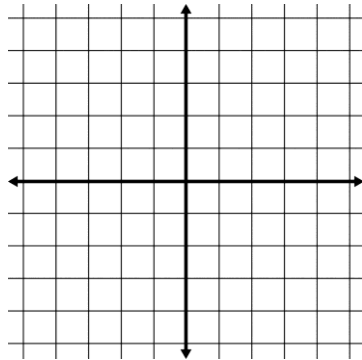
d) $g(x) = e^{2-2x}$

State whether the function is exponential growth or decay and describe its end behavior

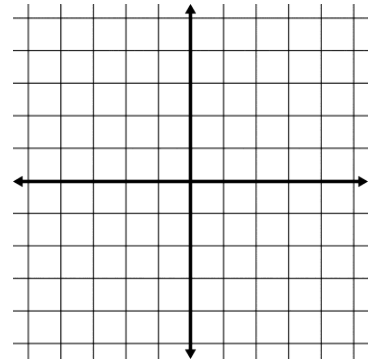
A) $f(x) = 2^{3x}$ B) $f(x) = 2^{-3x}$ C) $f(x) = \left(\frac{1}{4}\right)^x$ D) $f(x) = \left(\frac{1}{4}\right)^{-x}$

Graph the following functions on your calculator. Find the y-intercept and the horizontal asymptotes

$$f(x) = \frac{8}{1 + 3 \cdot 0.7^x}$$

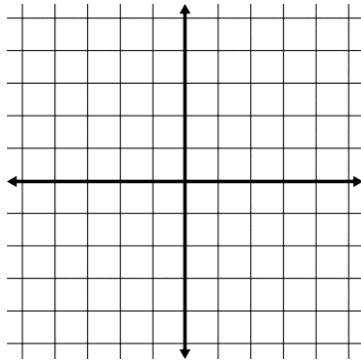


$$f(x) = \frac{20}{1 + 2 \cdot e^{-3x}}$$



Sketch a graph of the following functions

$$y = 2 \cdot 0.4^x$$



1) Determine the domain and range

2) Is the function even, odd or undefined for $x < 0$

3) Intervals of Increase or Decrease

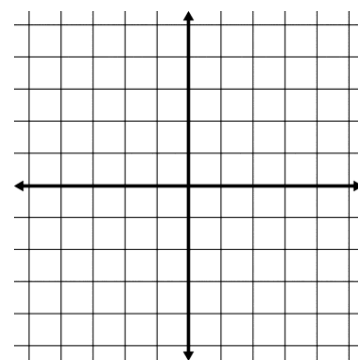
4) Find any extrema.

5) Determine the end behavior

6) Find any asymptotes

7) Intervals of Concavity

$$y = 3e^{-x}$$



1) Determine the domain and range

2) Is the function even, odd or undefined for $x < 0$

3) Intervals of Increase or Decrease

4) Find any extrema.

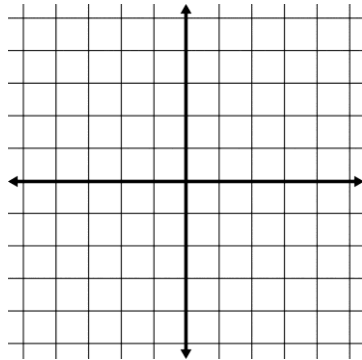
5) Determine the end behavior

6) Find any asymptotes

7) Intervals of Concavity

Sketch a graph of the following functions

$$y = \frac{4}{1 + 2e^{-x}}$$



1) Determine the minimum and Maximum capacity (Horizontal Asy)

2) Determine the y-intercept

3) Determine the domain and range

4) Intervals of Increase or Decrease

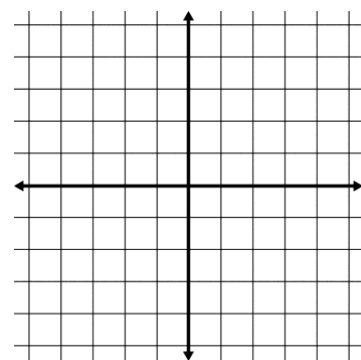
5) Determine the end behavior

6) Find any asymptotes

7) Determine Half the max capacity

8) Intervals of Concavity

$$y = \frac{4}{1 + 2e^x}$$



1) Determine the minimum and Maximum capacity (Horizontal Asy)

2) Determine the y-intercept

3) Determine the domain and range

4) Intervals of Increase or Decrease

5) Determine the end behavior

6) Find any asymptotes

7) Determine Half the max capacity

8) Intervals of Concavity

Using the data in the table below and assuming the growth is exponential, answer the following questions?

City	2000 Population	2007 Population
Austin, TX	465,622	656,562
San Jose, CA	898,759	939,899

a) When will the population of San Jose, California, surpass 1 million persons?

b) In what year will the population of Austin, TX and San Jose, CA be the same?

Based on recent Census Data, a logistic model for the population of Dallas, TX, t years after 1900, is as follows:

$$y = \frac{1301642}{1 + 21.602e^{-0.05054x}}$$

- a) What was the population of Dallas, TX in the year 2000?
- b) According to the model, what is Dallas' maximum sustainable population?
- c) According to this model, when was the population 1 million.

Bacteria Growth

The number of bacteria after t hours is given by

$$y = 150 e^{0.52t}$$

- a) What was the initial amount of bacteria present?

- b) How many bacteria are present after 4 hours?

- c) How many hours will it take until there are 400 bacteria?

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Chapter 3: Exponential, Logistic, and Logarithmic Functions

3.2: Exponential and Logistic Modeling

Tell whether the function is an exponential growth or decay function.
Then find the constant percentage rate of growth or decay.

1. $y = 10 \cdot (.7)^t$

2. $y = 5 \cdot (1.4)^t$

3. $y = 898 \cdot (1.0064)^t$

4. $y = 1203 \cdot (0.9858)^t$

Determine the exponential function that satisfies the given conditions

A) Initial Value 12, increasing at the rate of 8% per year

B) Initial Value 12, decreasing at the rate of 8% per year

C) Initial Population 200, tripling every 5 days

D) Initial Mass 20 grams, cutting in fourths once every 3 days

Determine a formula for the exponential function whose values are given
20)

x	g(x)
-2	-9.0625
-1	-7.25
0	-5.8
1	-4.64
2	-3.7123

Determine a formula for the exponential function whose points are given

21) (0, 4) (5, 8.05)

Find the logistic function that satisfies the given conditions

A) Initial Value 6: Max Capacity (Limit to growth) = 30
Passing through (1, 15)

B) Initial Population = 20, Max Capacity (Limit to growth) = 100
Passing through (4, 75)

32. Exponential Growth: The population of River City in the year 1910 was 4200. Assume the population increased at a rate of 2.25% per year.

a) Estimate the population in 1930.

b) Predict when the population reached 20,000.

Example 4: Suppose the half-life of a certain radioactive substance is 20 days and there are 5 grams present initially.

A) Express the amount of the substance remaining as function of time.

B) Find the time when there will be 1 gram of the substance remaining.

Watauga High School has 1200 students. Bob, Carol, Ted and Alice start a rumor, which spreads logistically so that

$S(t) = \frac{1200}{1 + 39e^{-0.9t}}$ models the number of students who have heard the rumor by the end of day t .

A) How many students have heard the rumor by the end of Day 0.

B) How long does it take for 1000 students to hear the rumor?

Use the data in the table and exponential regression to predict Dallas, TX population in 2015.

1950	434,462
1960	679,684
1970	844,401
1980	904,599
1990	1,006,877
2000	1,188,589

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Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.3: Logarithmic Functions and their graphs

What you'll Learn About

Changing between
Logarithmic and
exponential form:

If $x > 0$, $b > 0$ and
 $b \neq 1$, then
 $y = \log_b x$ if and only if
 $b^y = x$

Properties:

If $x > 0$, $b > 0$, $b \neq 1$, and
any real number y

- $\log_b 1 = 0$ because $b^0 = 1$
- $\log_b b = 1$ because $b^1 = b$
- $\log_b b^y = y$ because $b^y = b^y$
- $b^{\log_b x} = x$ because

$$\log_b x = \log_b y$$

Find the inverse function for $y = 2^x$

Evaluate the logarithmic expression without using a calculator

a) $\log_2 8 =$

b) $\log_3 \sqrt{3} =$

c) $\log_5 \frac{1}{25} =$

d) $\log_4 1 =$

e) $\log_7 7 =$

Evaluate the logarithmic expression without using a calculator

a) $\log 100 =$

b) $\log \sqrt[5]{10} =$

c) $\log \frac{1}{100} =$

d) $\ln \sqrt{e} =$

e) $\ln e^5 =$

f) $\ln \sqrt[5]{e} =$

Evaluate the logarithmic expression without using a calculator

a) $6^{\log_6 11} =$

b) $10^{\log 6} =$

c) $e^{\ln 4} =$

Use a calculator to evaluate the logarithmic expression if it is defined and check your result by evaluating the corresponding exponential expression

a) $\log 34.5 =$

b) $\log 0.43 =$

c) $\log (-3) =$

d) $\ln 23.5 =$

e) $\ln 0.48 =$

f) $\ln(-5) =$

Solve the equation

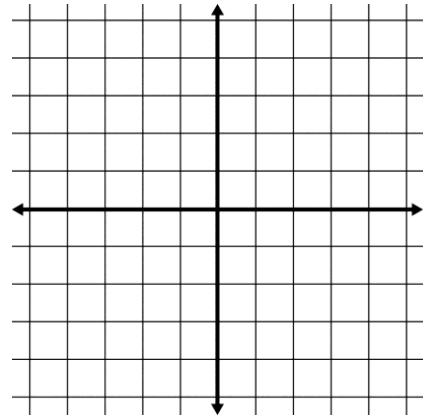
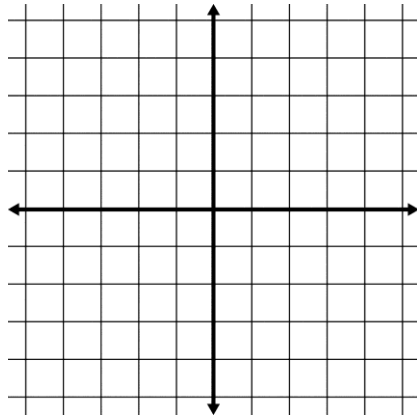
a) $\log x = 3$

b) $\log_2 x = 5$

Describe how to transform the graph of $y = \ln x$ into the graph of the given function. Sketch the graph by hand.

a) $g(x) = \ln(x + 2) + 1$

b) $h(x) = \ln(3 - x)$



1) Determine the vertical asymptotes

1) Determine the vertical asymptotes

2) Determine the x-intercept

2) Determine the x-intercept

3) Determine the domain and range

3) Determine the domain and range

4) Intervals of Increase or Decrease

4) Intervals of Increase or Decrease

5) Determine the end behavior

5) Determine the end behavior

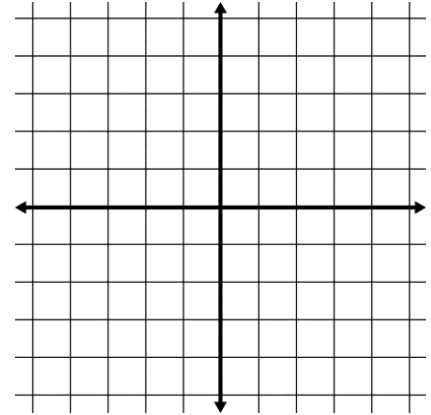
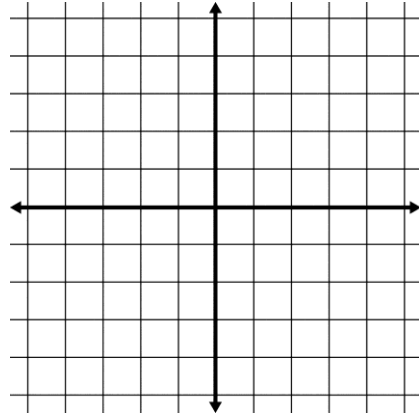
6) Intervals of Concavity

6) Intervals of Concavity

Describe how to transform the graph of $y = \ln x$ into the graph of the given function. Sketch the graph by hand.

a) $g(x) = -3\log x$

b) $h(x) = \log(-x) - 2$



1) Determine the vertical asymptotes

1) Determine the vertical asymptotes

2) Determine the x-intercept

2) Determine the x-intercept

3) Determine the domain and range

3) Determine the domain and range

4) Intervals of Increase or Decrease

4) Intervals of Increase or Decrease

5) Determine the end behavior

5) Determine the end behavior

6) Intervals of Concavity

6) Intervals of Concavity

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Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.4: Properties of Logarithmic Functions

What you'll Learn About

Use your Calculator to Determine which of the following are True.

1. $\log(5 + 2) = \log 5 + \log 2$

2. $\log(5 \cdot 2) = \log 5 + \log 2$

3. $\log(5 - 2) = \log 5 - \log 2$

4. $\log\left(\frac{5}{2}\right) = \log 5 - \log 2$

5. $\log(5 \cdot 2) = 2\log 5$

6. $\log\left(\frac{5}{2}\right) = \frac{\log 5}{\log 2}$

7. $\log(5^2) = \log 5 \cdot \log 5$

8. $\log(5^2) = 2\log 5$

9. $\ln(x + 2) = \ln x + \ln 2$

10. $\log(7x) = 7\log x$

11. $\log(5x) = \log 5 + \log x$

12. $\ln\left(\frac{x}{5}\right) = \ln x - \ln 5$

13. $\log\left(\frac{x}{4}\right) = \frac{\log x}{\log 4}$

14. $\log_4 x^3 = 3\log_4 x$

15. $\ln(x^2) = \ln x \cdot \ln x$

16. $\log|4x| = \log 4 + \log|x|$

Let b , R , and S are positive real numbers with $b \neq 1$, and c any real number

$$\bullet \log_b(RS) = \log_b R + \log_b S$$

$$\bullet \log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$$

$$\bullet \log_b R^c = c \log_b R$$

Prove the Product Rule for Logarithms: $\log_b(RS) = \log_b R + \log_b S$

Let $x = \log_b R$ and $y = \log_b S$

Assuming x and y are positive, use properties of logarithms to write the expression as a **sum or difference** of logarithms or multiples of logarithms

$$A) \log(8x)$$

$$B) \ln\left(\frac{5}{x}\right)$$

$$C) \log_2(x^5)$$

$$D) \log(8x^2y^4)$$

$$E) \ln\left(\frac{\sqrt{x^2 + 5}}{\sqrt[3]{x^4}}\right)$$

Let b , R , and S are positive real numbers with $b \neq 1$, and c any real number

$$\bullet \log_b(RS) = \log_b R + \log_b S$$

$$\bullet \log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$$

$$\bullet \log_b R^c = c \log_b R$$

Assuming x , y and z are positive, use properties of logarithms to write the expression as a **single** logarithm

$$A) \log x + \log 6$$

$$B) \ln x - \ln 6$$

$$C) \frac{1}{4} \log x$$

$$D) 6 \log x - \frac{1}{2} \log y$$

$$E) 5 \log(x^2 y) + 3 \log(y^2 z)$$

$$F) \ln x^5 - 2 \ln(xy)$$

$$\log_{64} 4096 = \frac{\log_8 4096}{\log_8 64} = \frac{4}{2}$$

because

$$8^y = 4096 \rightarrow 8^y = 8^4 \quad y = 4$$

$$8^y = 64 \rightarrow 8^y = 8^2 \quad y = 2$$

$$\log_{64} 4096 = \frac{\log_4 4096}{\log_4 64} = \frac{6}{3}$$

because

$$4^y = 4096 \rightarrow 4^y = 4^6 \quad y = 6$$

$$4^y = 64 \rightarrow 4^y = 4^3 \quad y = 3$$

For positive
real numbers a, b and x
with $a \neq 1$ and $b \neq 1$

$$\log_b x = \frac{\log_a x}{\log_a b} = \frac{\ln x}{\ln b}$$

Rewrite the following as an exponential function then solve for y

A) $y = \log_4 7$

Use the change of base formula and your calculator to evaluate the
logarithm

A) $\log_3 16$

B) $\log_{1/2} 2$

Express using only natural logarithms

A) $g(x) = \log_5 x$

B) $g(x) = \log_2(x + y)$

Describe the transformation of each function from the original function $\ln(x)$ or $\log(x)$

42) $f(x) = \ln(x) + 2$

44) $f(x) = \ln(-x) + 2$

46) $f(x) = \ln(5 - x)$

A) $f(x) = \ln(x - 5)$

52) $f(x) = -3\log(1-x) + 1$

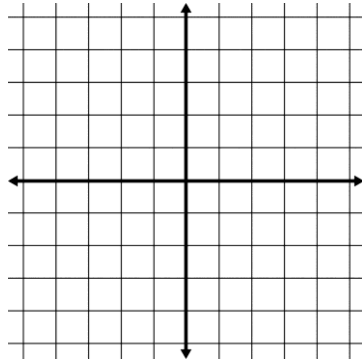
B) $y = \frac{1}{3}\log x$

C) $y = \log(2x)$

D) $y = \log\left(\frac{1}{2}x\right)$

Sketch a graph of the following functions

$$f(x) = \log_3(5x)$$



1) Determine the vertical asymptotes

2) Determine the x-intercept

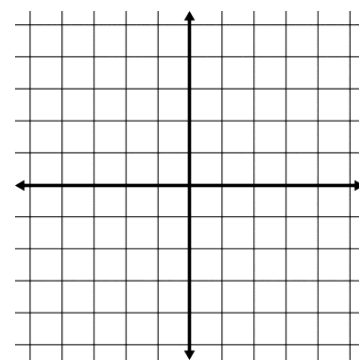
3) Determine the domain and range

4) Intervals of Increase or Decrease

5) Determine the end behavior

6) Intervals of Concavity

$$f(x) = \ln(x^4)$$



1) Determine the vertical asymptotes

2) Determine the x-intercept

3) Determine the domain and range

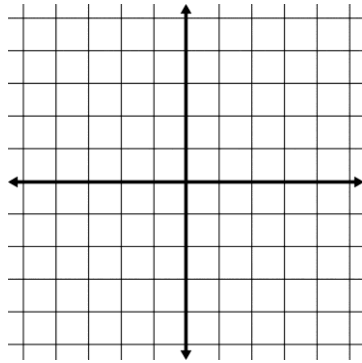
4) Intervals of Increase or Decrease

5) Determine the end behavior

6) Intervals of Concavity

Sketch a graph of the following functions

$$f(x) = \log_3(x - 4)$$



1) Determine the vertical asymptotes

2) Determine the x-intercept

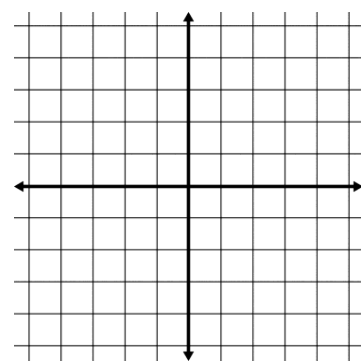
3) Determine the domain and range

4) Intervals of Increase or Decrease

5) Determine the end behavior

6) Intervals of Concavity

$$f(x) = \ln(4 - x)$$



1) Determine the vertical asymptotes

2) Determine the x-intercept

3) Determine the domain and range

4) Intervals of Increase or Decrease

5) Determine the end behavior

6) Intervals of Concavity

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Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.5: Equation Solving and Modeling

What you'll Learn About

Find the exact solution algebraically, and check it by substituting into the original equation.

A) $\left(\frac{1}{4}\right)^x = \frac{1}{16}$

B) $20\left(\frac{1}{2}\right)^{x/3} = 5$

C) $2(3)^{x/2} = 6$

D) $2(3)^{-x/2} = 54$

E) $\log x = 5$

F) $\log_2(x - 4) = 3$

Solve each equation algebraically

A) $2 \cdot 3^x = 5$

B) $50(e)^{0.03x} = 500$

C) $2\ln(x+3)+6=10$

D) $2 - \log(x+3) = 10$

Solve each equation.

A) $\log x^3 = 9$

B) $\frac{3^x - 3^{-x}}{2} = 5$

C) $3e^{2x} + 4e^x - 4 = 0$

D)
$$\frac{700}{1 + 20e^{0.2x}} = 200$$

$$E) \quad \frac{1}{2} \ln(x+2) - \ln(x) = 0$$

$$F) \quad \log(x+2) + \log(x+3) = 4\log 2$$

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