

Using the data in the table below and assuming the growth is exponential, answer the following questions?

City	2000 Population	2007 Population
Austin, TX	465,622	656,562
San Jose, CA	898,759	939,899

$$SJ \quad y = 898,759(1.006)^x$$

a) When will the population of San Jose, California, surpass 1 million persons?

$$1,000,000 = 898,759(1.006)^x$$

$$1.112 = 1.006^x$$

$$\ln 1.112 = \ln 1.006^x$$

$$\ln 1.112 = x \ln 1.006$$

$$x = \frac{\ln 1.112}{\ln 1.006}$$

$$\approx 17.746$$

18 yrs after 2000

or 2018

b) In what year will the population of Austin, TX and San Jose, CA be the same?

Based on recent Census Data, a logistic model for the population of Dallas, TX,  $t$  years after 1900, is as follows:

$$y = \frac{1301642}{1 + 21.602e^{-0.05054x}}$$

- a) What was the population of Dallas, TX in the year 2000?
- b) According to the model, what is Dallas' maximum sustainable population?
- c) According to this model, when was the population 1 million.

$$1,000,000 \leftarrow \frac{1301642}{1 + 21.602e^{-0.05054x}}$$

$$1 + 21.602e^{-0.05054x} = \frac{1301642}{1,000,000}$$

$$1 + 21.602e^{-0.05054x} = 1.301642$$

$$\frac{21.602e^{-0.05054x}}{21.602} = \frac{.301642}{21.602}$$

$$e^{-0.05054x} = .01396$$

$$\ln e^{-0.05054x} = \ln(.01396)$$

$$-.05054x = \ln(.01396)$$

$$x = \frac{\ln(.01396)}{-.05054}$$

$$\approx \cancel{276}$$

$$= 84.518$$

### Bacteria Growth

The number of bacteria after  $t$  hours is given by

$$y = 150e^{0.521t}$$

- a) What was the initial amount of bacteria present?
- b) How many bacteria are present after 4 hours?
- c) How many hours will it take until there are 400 bacteria?

$$\frac{400}{150} \quad 400 = 150e^{.521t}$$

$$\frac{40}{15} = \frac{8}{3} \quad \frac{8}{3} = e^{.521t}$$

$$\ln \frac{8}{3} = .521t$$

$$t = \frac{\ln \frac{8}{3}}{.521} \approx 1.883 \text{ hours}$$

Watauga High School has 1200 students. Bob, Carol, Ted and Alice start a rumor, which spreads logistically so that

$S(t) = \frac{1200}{1 + 39e^{-0.9t}}$  models the number of students who have heard the rumor by the end of day  $t$ .

A) How many students have heard the rumor by the end of Day 0.

B) How long does it take for 1000 students to hear the rumor?

Use the data in the table and exponential regression to predict Dallas, TX population in 2015.

1950	434,462
1960	679,684
1970	844,401
1980	904,599
1990	1,006,877
2000	1,188,589

Solve each equation.

A)  $\log x^3 = 9$

$$\sqrt[3]{10^9} = \sqrt[3]{x^3}$$

$$x = \sqrt[3]{10^9} = (10^9)^{1/3} = 10^{9/3} = 10^3$$

B)  $\frac{3^x - 3^{-x}}{2} = 5$

$$e^x = y$$

$$3y^2 + 4y - 4$$

$$(3y-2)(y+2) = 0$$

$$3y-2=0 \quad y+2=0$$

C)  $3e^{2x} + 4e^x - 4 = 0$

$$(3e^x - 2)(e^x + 2) = 0$$

$$3e^x - 2 = 0 \quad \left\{ \begin{array}{l} e^x + 2 = 0 \\ e^x = -2 \end{array} \right.$$

$$\frac{3e^x}{3} = \frac{2}{3}$$

$$e^x = \frac{2}{3}$$

$$\ln e^x = \ln \frac{2}{3}$$

$$e^x = -2$$

$$\ln e^x = \ln -2$$

$$x \ln e = \ln -2$$

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$$x \ln e = \ln \frac{2}{3}$$

$$x = \ln \frac{2}{3}$$

$$\approx -.405$$

$$x = \ln -2$$

No solution

extraneous solution

$$\frac{\frac{5}{2} \cdot \frac{1}{20}}{\frac{1}{-}} = \frac{5}{40} = \frac{1}{8}$$

$$D) \left( \cancel{1+20e^{2x}} \right) \left( \frac{700}{\cancel{1+20e^{2x}}} = 200 \right) (1+20e^{2x})$$

$$\frac{700}{200} = \frac{200(1+20e^{2x})}{200}$$

$$\frac{7}{2} = 1 + 20e^{2x}$$

$$\frac{5}{2} = \frac{20e^{2x}}{20}$$

$$\frac{1}{8} = e^{2x}$$

$$\ln\left(\frac{1}{8}\right) = \ln e^{2x}$$

$$\ln\left(\frac{1}{8}\right) = 2x \ln e$$

$$\ln\left(\frac{1}{8}\right) = 2x$$

$$x = \frac{\ln\left(\frac{1}{8}\right)}{2}$$

$$x \approx -10.397$$

$$\frac{1}{2} \ln(4) - \ln 2$$

$$\ln 4^{1/2} - \ln 2$$

$$\ln 2 - \ln 2$$

$$\frac{1}{2} \ln(-1+2) - \ln(-1)$$

$$E) \quad \frac{1}{2} \ln(x+2) - \ln(x) = 0$$

$$\ln(x+2)^{1/2} - \ln x = 0$$

$$\ln \frac{(x+2)^{1/2}}{x} = 0$$

$$e^0 = \frac{(x+2)^{1/2}}{x}$$

$$1 = \frac{(x+2)^{1/2}}{x}$$

$$(x)^2 = (\sqrt{x+2})^2$$

$$x^2 = x+2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad \cancel{x = -1}$$

↑  
extraneous

$$F) \quad \log(x+2) + \log(x+3) = 4 \log 2$$

$$\log(x+2) + \log(x+3) - (4 \log 2) = 0$$

$$\log(x+2) + \log(x+3) - \log 2^4 = 0$$

$$\log \frac{(x+2)(x+3)}{16} = 0$$

$$\log \frac{x^2 + 5x + 6}{16} = 0$$

$$10^0 = \frac{x^2 + 5x + 6}{16}$$

$$1 = \frac{x^2 + 5x + 6}{16}$$

$$16 = x^2 + 5x + 6$$

$$0 = x^2 + 5x - 10$$

$$\frac{-5 \pm \sqrt{65}}{2}$$

$$x = \frac{-5 + \sqrt{65}}{2} \approx 1.531$$

$$x = \frac{-5 - \sqrt{65}}{2} \approx -6.531$$

↑  
extraneous

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