PRE-CALCULUS: by Finney, Demana, Watts and Kennedy
Chapter 3: Exponential, Logistic, and Logarithmic Functions
3.1: Exponential and Logistic Functions

Exponential Function
A function that can be rewritten in the form $y = a \cdot b^x$, where a is non-zero, b is positive, and $b \neq 1$.
a: initial value at x= 0
b: base

Which of the following are exponential functions?
For those that are exponential functions, state the initial value and the base. For those that are not, explain.

$f(x) = 1 \cdot 3^x$

A) $f(x) = 3^x$

Yes exponential

$f(0) = 3^0 = 1$

base : 3

B) $g(x) = 6x^{-4}$

No Variable not exponent

C) $h(x) = -2 \cdot 1.5^x$

Yes exponential ↙ Initial Value

$f(0) = -2 \cdot 1.5^0 = -2$

Base 1.5

D) $h(x) = 7 \cdot -2^x$

No b-value is Negative

E) $f(x) = 5 \cdot 6^{\pi}$

No Constant Function No variable

Compute the exact value of the function without using a calculator

Negative exponent take Reciprocal

$x^{\frac{m \to \text{Power}}{n \to \text{Root}}}$

A) $2 \cdot 4^x$ when x = 0

$2 \cdot 4^0$

$2 \cdot 1$

2

B) $2 \cdot 4^x$ when x = -3

$2 \cdot 4^{-3}$

$2 \cdot \frac{1}{4^3}$

$2 \cdot \frac{1}{64} = \frac{1}{32}$

C) $-2 \cdot 4^x$ when x = 1/2

$-2 \cdot 4^{1/2}$

$-2 \cdot \sqrt{4}$

$-2 \cdot 2$

-4

D) $3 \cdot 8^x$ when x = -2/3

$3 \cdot 8^{-2/3}$

$3 \cdot \frac{1}{8^{2/3}}$

$3 \cdot \frac{1}{(\sqrt[3]{8})^2}$

$3 \cdot \frac{1}{4} = \frac{3}{4}$

$f(x) = a \cdot b^x$ a: Inital Value	Determine a formula for the exponential function g(x) and h(x) whose values are given in the table

x	g(x)		x	h(x)
-2	4/9		-2	128
-1	4/3		-1	32
0	4		0	8
1	12		1	2
2	36		2	1/2

$g(x) = a \cdot b^x \Rightarrow g(x) = 4 \cdot 3^x$

$a = 4$

$g(x) = 4 \cdot b^x$

$12 = 4 \cdot b^1$

$12 = 4b$

$b = 3$

$a = 8$

$h(x) = 8 \cdot b^x$

$2 = 8 \cdot b^1$

$\frac{2}{8} = \frac{8b}{8}$

$h(x) = 8 \cdot \left(\frac{1}{4}\right)^x$

$b = \frac{1}{4}$

Given 2 points on the graph of an exponential function, find the formula

A) (0, 2) (2, 18)

$x \quad f(x)$

$a = 2$

$f(x) = ab^x$

$f(x) = 2 \cdot b^x$

$\frac{18}{2} = \frac{2 \cdot b^2}{2}$

$9 = b^2$

$b = 3$

$f(x) = 2 \cdot 3^x$

$\cancel{\frac{\frac{3}{e}}{3}} \cdot \frac{1}{\cancel{3}}$

B) (0, 3) $\left(3, \frac{3}{e}\right)$

$x \quad f(x)$

$a = 3$

$f(x) = ab^x$

$f(x) = 3 \cdot b^x$

$\frac{\frac{3}{e}}{3} = \frac{3 \cdot b^3}{3}$

$\sqrt[3]{\frac{1}{e}} = \sqrt[3]{b^3}$

$\frac{1}{\sqrt[3]{e}} = b$

$f(x) = 3 \cdot \left(\frac{1}{\sqrt[3]{e}}\right)^x$

$\frac{1}{\sqrt[3]{e}} = \frac{1}{e^{1/3}}$

$= e^{-1/3}$

$3 \cdot \left(e^{-\frac{1}{3}}\right)^x$

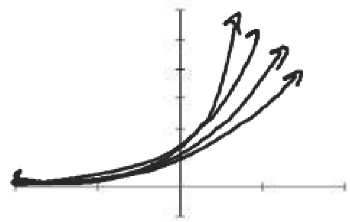
$3 \cdot e^{-\frac{x}{3}}$

Exponential Growth
$b > 1$

Exponential Decay
$0 < b < 1$

Sketch a graph of the following functions in the same viewing window [-2, 2] [-1, 6]

$y = 2^x \quad y = 3^x \quad y = 4^x \quad y = 5^x$



1) Determine the domain and range
D: $(-\infty, \infty)$
R: $(0, \infty)$

2) Is the function even, odd or neither
Neither

3) Intervals of Increase or Decrease
Inc $(-\infty, \infty)$

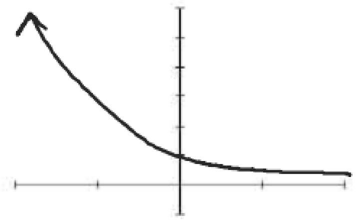
4) Find any extrema.
None

5) Determine the end behavior
$\lim_{x \to \infty} f(x) = \infty \qquad \lim_{x \to -\infty} f(x) = 0$

6) Find any asymptotes
H.A. $y = 0$

7) Intervals of Concavity
Concave up $(-\infty, \infty)$

$y = \left(\frac{1}{2}\right)^x \quad y = \left(\frac{1}{3}\right)^x \quad y = \left(\frac{1}{4}\right)^x \quad y = \left(\frac{1}{5}\right)^x$



1) Determine the domain and range
D: $(-\infty, \infty)$
R: $(0, \infty)$

2) Is the function even, odd or neither
Neither

3) Intervals of Increase or Decrease
Dec $(-\infty, \infty)$

4) Find any extrema.
None

5) Determine the end behavior
$\lim_{x \to \infty} f(x) = 0 \qquad \lim_{x \to -\infty} f(x) = \infty$

6) Find any asymptotes
H.A. $y = 0$

7) Intervals of Concavity
Concave up $(-\infty, \infty)$

Describe how to transform the graph of $\underline{f(x) = 2^x}$ into the graph of g

a) $g(x) = \underline{2^{x-1}}$

Shift Right 1

b) $g(x) = 2^{-x}$

Reflect over y-axis

$1 \cdot 2^{3-x}$

c) $g(x) = \underline{3} \cdot 2^x$

Vertical Stretch by factor of 3

d) $g(x) = 2^{3-x}$

$= 2^{-x+3}$

$2^{-(x-3)}$

Reflect over y-axis

Right 3

Describe how to transform the graph of $\mathbf{f(x) = e^x}$ into the graph of g

a) $g(x) = e^{4x}$

Horizontal Compression by factor of $\frac{1}{4}$

b) $g(x) = e^{-4x}$

Reflection over y-axis

Horizontal compression by factor of $\frac{1}{4}$

c) $g(x) = 3 \cdot e^x + 1$

Vertical stretch by factor of 3

up 1

d) $g(x) = e^{2-2x}$

$-2x+2$

e^{-2x+2}

$e^{-2(x-1)}$

Reflect y-axis

H.C. by factor $\frac{1}{2}$

Right 1

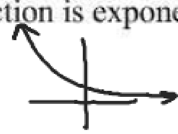
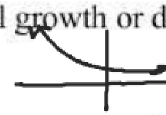
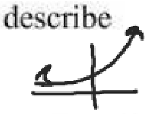
$y = a b^x$

$y = 3 \cdot 2^x$

$y = e^{4x}$

$y = 1 \cdot e^{4x}$

State whether the function is exponential growth or decay and describe its end behavior

A) $f(x)=2^{3x}$ B) $f(x)=2^{-3x}$ C) $f(x)=\left(\frac{1}{4}\right)^{x}$ D) $f(x)=\left(\frac{1}{4}\right)^{-x}$

A) Growth

$\lim_{x\to\infty} f(x)=\infty$

$\lim_{x\to-\infty} f(x)=0$

B) $=\frac{1}{2^{3x}}$

Decay

$\lim_{x\to\infty} f(x)=0$

$\lim_{x\to-\infty} f(x)=\infty$

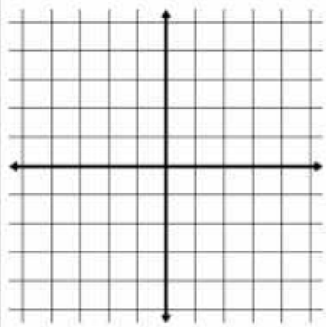
C) Decay

D) $=4^{x}$

Growth

Graph the following functions on your calculator. Find the y-intercept and the horizontal asymptotes

$f(x)=\dfrac{8}{1+3\cdot 0.7^{x}}$



$f(x)=\dfrac{20}{1+2\cdot e^{-3x}}$