Using the IVT to determine of a function has a zero on a given interval

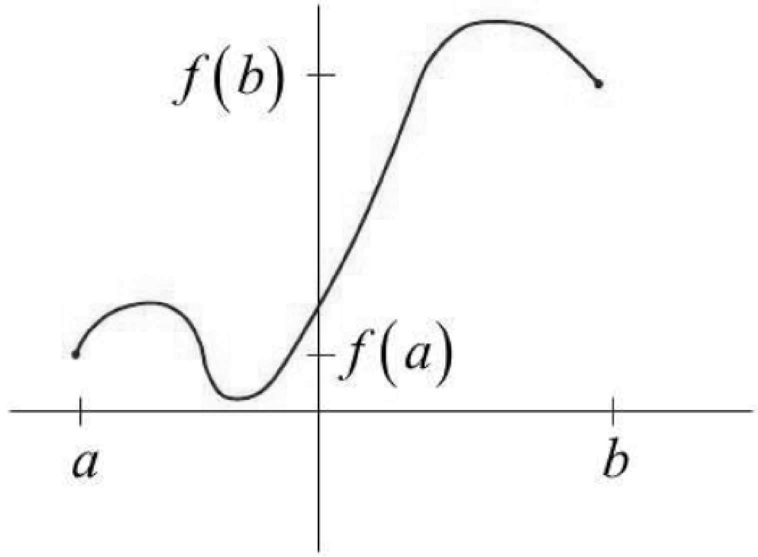
If $f(a) < 0$ and $f(b) > 0$ we can conclude there is at least one zero in the interval [a,b]

Or

If $f(a) > 0$ and $f(b) < 0$ we can conclude there is at least one zero in the interval [a,b]

Intermediate Value Theorem:

If a function is continuous between a and b, then it takes on every value between f(a) and f(b)



Because the function is continuous, it must take on every y value between f(a) and f(b)

1. Determine if $f(x) = 3x^2 + 4x - 4$ crosses the x-axis between $x = -5$ and $x = 0$.

2. Determine if $f(x) = 3x^2 + 4x - 4$ crosses the x-axis between $x = 1$ and $x = 5$.

What you'll Learn About

Divide f(x) by d(x) using factoring.

1) $f(x) = x^2 + 5x + 6$ $d(x) = x + 2$

$$\frac{x^2+5x+6}{x+2} \qquad \frac{\cancel{(x+2)}(x+3)}{\cancel{x+2}} = x+3$$

Divide f(x) by d(x) using long division.

2) $f(x) = x^2 + 5x + 6$ $d(x) = x + 2$

$$\begin{array}{r} x + 3 \\ x+2 \overline{) x^2 + 5x + 6} \\ -\underline{(x^2 + 2x)} \\ 3x + 6 \\ -\underline{(3x + 6)} \\ 0 \end{array}$$

$x+3$

Divide f(x) by d(x) using synthetic division.

3) $f(x) = x^2 + 5x + 6$ $d(x) = x + 2$

$x+2=0$
$x=-2$

$$\begin{array}{r|rrr} -2 & 1 & 5 & 6 \\ & & -2 & -6 \\ \hline & 1 & 3 & \boxed{0} \end{array}$$

$x+3$

	Divide f(x) by d(x) by using long division, and write a summary statement in polynomial form and fraction form. 1) $f(x) = 3x^3 + 5x^2 + 8x + 7 \quad d(x) = 3x + 2$ $$\begin{array}{r} x^2 + x + 2 \\ 3x+2 \overline{\smash{)}\, 3x^3 + 5x^2 + 8x + 7} \\ (-)\, \underline{3x^3 + 2x^2} \\ 3x^2 + 8x \\ (-)\, \underline{3x^2 + 2x} \\ 6x + 7 \\ \underline{-\, 6x + 4} \\ 3 \end{array}$$ $$x^2 + x + 2 + \frac{3}{3x+2}$$ Divide f(x) by d(x) by using synthetic division, and write a summary statement in polynomial form and fraction form. 2) $f(x) = 3x^3 + 5x^2 + 8x + 7 \quad d(x) = 3x + 2$ $3x+2=0$ $x = \frac{-2}{3}$ $$\begin{array}{c\|cccc} -\frac{2}{3} & 3 & 5 & 8 & 7 \\ & & -2 & -2 & -4 \\ \hline & 3 & 3 & 6 & \boxed{3} \end{array}$$ $$\frac{3x^2 + 3x + 6}{3} + \frac{3}{3x+2}$$ $$x^2 + x + 2 + \frac{3}{3x+2}$$

	Divide f(x) by d(x) and write a summary statement in polynomial form and fraction form. 1) $f(x) = 2x^3 - 3x^2 - 5x - 12 \quad d(x) = x - 3$ $$\begin{array}{r\|rrrr} 3 & 2 & -3 & -5 & -12 \\ & & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & \boxed{0} \end{array}$$ $2x^2 + 3x + 4$ $$\begin{array}{r} 2x^2 + 3x + 4 \\ x-3 \overline{) 2x^3 - 3x^2 - 5x - 12} \\ (-)\underline{2x^3 - 6x^2} \\ 3x^2 - 5x \\ (-)\underline{3x^2 - 9x} \\ 4x - 12 \\ -\underline{4x - 12} \\ 0 \end{array}$$ Divide f(x) by d(x) and write a summary statement in polynomial form and fraction form. 1) $f(x) = 2x^4 - x^3 - 2 \quad d(x) = 2x^2 + x + 1$ $$\begin{array}{r} x^2 - x \\ 2x^2+x+1 \overline{) 2x^4 - x^3 + 0x^2 + 0x - 2} \\ (-)\underline{2x^4 + x^3 + x^2} \\ -2x^3 - x^2 + 0x \\ (-)\underline{-2x^3 - x^2 - x} \\ x - 2 \end{array}$$ $x^2 - x + \frac{x-2}{2x^2+x+1}$

	Use the Remainder Theorem to find the remainder when f(x) is divided by x – k a) $f(x) = 3x^2 + 7x - 20$ $k = 2$ $f(2) = 3(2)^2 + 7(2) - 20$ $12 + 14 - 20$ 6 $\begin{array}{r\|rrr} 2 & 3 & 7 & -20 \\ & & 6 & 26 \\ \hline & 3 & 13 & \boxed{6} \end{array}$ b) $f(x) = 3x^2 + 7x - 20$ $k = -1$ $\begin{array}{r\|rrr} -1 & 3 & 7 & -20 \\ & & -3 & -4 \\ \hline & 3 & 4 & \boxed{-24} \end{array}$ c) $f(x) = 3x^2 + 7x - 20$ $k = -4$ $\begin{array}{r\|rrr} -4 & 3 & 7 & -20 \\ & & -12 & 20 \\ \hline & 3 & -5 & \boxed{0} \end{array}$ d) $f(x) = x^4 - 1$ $k = 1$ $\begin{array}{r\|rrrrr} 1 & 1 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & \boxed{0} \end{array}$

Factor

Remainder is zero!

Use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

A) $x - 2$; $x^3 - 4x^2 + 8x - 8$

X-2 is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & \boxed{0} \end{array}$$

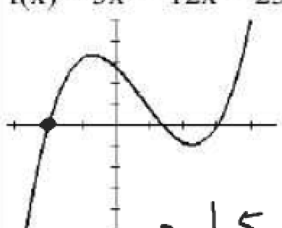
B) $x + 3$; $x^3 + 2x^2 - 4x - 2$

X+3 is not a Factor

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -4 & -2 \\ & & -3 & 3 & 3 \\ \hline & 1 & -1 & -1 & \boxed{1} \end{array}$$

Use the graph in number 26 to guess possible linear factors of f(x). Then completely factor f(x) with the aid of synthetic division.

$f(x) = 5x^3 - 12x^2 - 23x + 42$



$x = -2 \quad x = \frac{3}{2} \quad x = 3$

$(x+2)(x-\frac{3}{2})(x-3)$

$(x+2)(2x-3)(x-3)$

$$\begin{array}{r|rrrr} -2 & 5 & -12 & -23 & 42 \\ & & -10 & 44 & -42 \\ \hline 3 & 5 & -22 & 21 & \boxed{0} \\ & & 15 & -21 & \\ \hline & 5 & -7 & \boxed{0} & \end{array}$$

$5x^2 - 22x + 21$

$5x - 7 = 0$

$(x+2)(x-3)(5x-7)$