Find the polynomial function with leading coefficient 4 that has the given degree and zeros.

A) Degree 3,with 2, -1, and 4 as zeros

$x=2 \quad x=-1 \quad x=4$

$(x-2)(x+1)(x-4)$

$(x-2)(x^2-3x-4)$

x^3-3x^2-4x

$-2x^2+6x+8$

$4\left[x^3-5x^2+2x+8\right]$

$y = 4x^3-20x^2+8x+32$

B) Degree 3 with 5, 1/3, and 2/3 as zeros

$x=5 \quad x=\frac{1}{3} \quad x=\frac{2}{3}$

$\left(x-5\right)\left(x-\frac{1}{3}\right)\left(x-\frac{2}{3}\right)$

$(x-5)(3x-1)(3x-2)$

$(x-5)(9x^2-9x+2)$

$9x^3-9x^2+2x$

$-45x^2+45x-10$

$\frac{4}{9}\left(9x^3-54x^2+47x-10\right)$

$4x^3-24x^2+\frac{188}{9}x-\frac{40}{9}$

$3(x)=\left(\frac{1}{3}\right)^3$

$3x=1$

$3x-1=0$

$\frac{9x}{9}=\frac{4}{9}$

$x=\frac{4}{9}$

Using only algebraic methods, find the cubic function with the given table of values

x	-4	0	3	5
f(x)	0	180	0	0

(-4,0) (0,180) (3,0) (5,0)

$y = a(x+4)(x-3)(x-5)$

$180 = a(0+4)(0-3)(0-5)$

$60a = 180$

$a = 3$

$y = (x+4)(x^2 - 8x + 15)$

$x^3 - 8x^2 + 15x$

$+4x^2 - 32x + 60$

$3(x^3 - 4x^2 - 17x + 60)$

$3x^3 - 12x^2 - 51x + 180$

Finding Rational Zeros 1) List all possible rational zeros p/q where q is the leading coefficient and p is the constant 2) Use your calculator to find the zeros and then use synthetic division and algebra to prove that the zeros that you chose are rational zeros	Use the Rational Zeros Theorem to write a list of all potential rational zeros and then determine which ones, if any, are zeros. $f(x) = 3x^3 + 4x^2 - 5x - 2$ $q = \pm 1, \pm 3$ $p = \pm 1, \pm 2$ $\frac{p}{q} = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}$ $1 \mid 3 \;\; 4 \;\; -5 \;\; -2$ $\quad\quad 3 \;\; 7 \;\; 2$ $-2 \mid 3 \;\; 7 \;\; 2 \mid 0$ $\quad\quad -6 \;\; -2$ $\quad 3 \;\; 1 \mid 0$ $3x^2 + 7x + 2 = 0$ $3x + 1 = 0$ $x = -\frac{1}{3}$ $x = -2, -\frac{1}{3}, 1$ Use the Rational Zeros Theorem to write a list of all potential rational zeros and then determine which ones, if any, are zeros. $f(x) = x^3 - 3x^2 + 1$ $p: \pm 1$ $q: \pm 1$ $\frac{p}{q} = \pm 1$ $1 \mid 1 \;\; -3 \;\; 0 \;\; 1$ $\quad\quad 1 \;\; -2 \;\; -2$ $\quad 1 \;\; -2 \;\; -2 \mid -1$ $-1 \mid 1 \;\; -3 \;\; 0 \;\; 1$ $\quad\quad -1 \;\; 4 \;\; -4$ $\quad 1 \;\; -4 \;\; 4 \mid -3$ No Rational Zeros.

Find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

52. $f(x) = x^3 - 6x^2 + 7x + 4$

$p = 4 \quad \pm 1, \pm 2, \pm 4$

$q = 1 \quad \pm 1$

$\frac{p}{q} = \pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 7 & 4 \\ & & 4 & -8 & -4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$x^2 - 2x - 1$

$x = 4$

Find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

$f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$