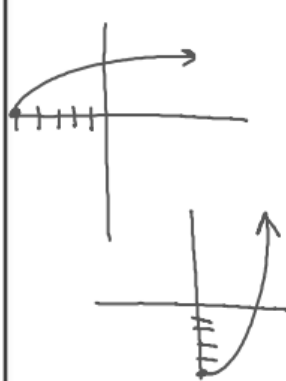


$f^{-1}(x) \rightarrow$  Inverse

- 1) Change  $f(x)$  to  $y$
- 2) Switch  $x$  and  $y$
- 3) Solve for  $y$

$f(x)$ Domain (x) Range (y)	$f^{-1}(x)$ Domain Range
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Find a formula for  $f^{-1}(x)$ . Give the domain of  $f^{-1}(x)$ , including any restrictions "inherited" from  $f$ .

A.  $f(x) = 5x + 2$   
 $D: (-\infty, \infty)$   $R: (-\infty, \infty)$   
 $f^{-1}(x) = \frac{1}{5}x - \frac{2}{5}$   
 $D: (-\infty, \infty)$

B.  $f(x) = \frac{3x+2}{x-1}$   
 $D: (-\infty, 1) \cup (1, \infty)$   $R: (-\infty, 3) \cup (3, \infty)$   
 $f^{-1}(x) = \frac{x+2}{x-3}$   
 $D: (-\infty, 3) \cup (3, \infty)$

C.  $f(x) = \sqrt{x+5}$   
 $D: [-5, \infty)$   $R: [0, \infty)$   
 $y = \sqrt{x+5}$   
 $(x+5) = y^2$   
 $x^2 = y+5$   
 $y = x^2 - 5$   
 $f^{-1}(x) = x^2 - 5$   
 $D: [0, \infty)$

D.  $f(x) = \sqrt[3]{x^3 - 2}$   
 $D: [-\infty, \infty)$   $R: [-\infty, \infty)$   
 $x^3 - 2 \geq 0 \Rightarrow x^3 \geq 2 \Rightarrow x \geq \sqrt[3]{2}$   
 $[ \sqrt[3]{2}, \infty )$   
 $f(x) = \sqrt{x^3 - 2}$   
 $R: [0, \infty)$   
 $y = \sqrt{x^3 - 2}$   
 $(x^3 - 2) = y^2$   
 $x^3 = y^2 + 2$   
 $y^3 = x^2 + 2$   
 $y = \sqrt[3]{x^2 + 2}$   
 $f^{-1}(x) = \sqrt[3]{x^2 + 2}$   
 $D: [0, \infty)$

E.  $f(x) = \sqrt[3]{2x+1}$   
 $D: (-\infty, \infty)$   $R: (-\infty, \infty)$   
 $y = \sqrt[3]{2x+1}$   
 $x = \sqrt[3]{2y+1}$   
 $x^3 = 2y+1$   
 $x^3 - 1 = 2y$   
 $y = \frac{x^3 - 1}{2}$   
 $f^{-1}(x) = \frac{x^3 - 1}{2}$   
 $D: (-\infty, \infty)$

$$f(g(x)) = g(f(x)) = x$$

Confirm that  $f$  and  $g$  are inverses by showing that  $f(g(x))$  and  $g(f(x)) = x$ .

A.  $f(x) = x^3 + 1$  and  $g(x) = \sqrt[3]{x-1}$

$$f(g(x))$$

$$\begin{aligned} f(\sqrt[3]{x-1}) &= (\sqrt[3]{x-1})^3 + 1 \\ &= x - 1 + 1 \\ &= x \end{aligned}$$

$$g(f(x)) =$$

$$\begin{aligned} g(x^3 + 1) &= \sqrt[3]{x^3 + 1 - 1} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

32)  $f(x) = \frac{x+3}{x-2}$  and  $g(x) = \frac{2x+3}{x-1}$

$$\frac{2x-3}{x-1} - 2$$

$$f(g(x)) =$$

$$f\left(\frac{2x+3}{x-1}\right) = \frac{\frac{2x+3}{x-1} + 3}{\frac{2x+3}{x-1} - 2}$$

$$\frac{\frac{2x+3}{x-1} + \frac{3(x-1)}{x-1}}{\frac{2x+3}{x-1} - \frac{2(x-1)}{x-1}}$$

$$\frac{2x+3 + 3x-3}{x-1} \div \frac{2x+3 - 2x+2}{x-1}$$

$$\frac{5x}{x-1} \cdot \frac{x-1}{5} = \frac{5x}{5} = x$$

$$g(f(x)) = \frac{2\left(\frac{x+3}{x-2}\right) + 3}{\frac{x+3}{x-2} - 1}$$

$$= \frac{\frac{2x+6}{x-2} + \frac{3(x-2)}{x-2}}{\frac{x+3}{x-2} - \frac{1(x-2)}{x-2}}$$

$$\frac{\frac{2x+6}{x-2} + \frac{3x-6}{x-2}}{\frac{x+3}{x-2} + \frac{-x+2}{x-2}}$$

$$\frac{\frac{5x}{x-2} \cdot \frac{x-2}{5}}{\frac{5}{x-2}} = \frac{5x}{5} = x$$

$$\frac{5x}{5} = x$$

$$\frac{5x}{5} = x$$

$$f(x) = \frac{x+3}{4}$$

$$g(x) = 4x-3$$

$$f(g(x)) =$$

$$f(4x-3) = \frac{x+3}{4}$$

$$= \frac{4x-3+3}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

$$g(f(x)) =$$

$$g\left(\frac{x+3}{4}\right) = 4x-3$$

$$= 4\left(\frac{x+3}{4}\right) - 3$$

$$= \frac{4x+3-3}{1}$$

$$f(x) = \sqrt{x+2}$$

$$y = \sqrt{x+2}$$

$$(x+2) = (\sqrt{y+2})^2$$

$$x+2 = y+2$$

$$y = x^2 - 2$$

$$f^{-1}(x) = x^2 - 2$$

$$D: [0, \infty)$$