What you will learn about:
Adding and Subtracting Rational Expressions
With Common Demoninators

Rational Expression Addition:
If p, q, and r are polynomials where $r \neq 0$, then

$$\frac{p}{r}+\frac{q}{r}=\frac{p+q}{r}$$

Add:

$\frac{5}{18}+\frac{7}{18} = \frac{12}{18} = \frac{2}{3}$

$\frac{7}{16}+\frac{5}{16} = \frac{12}{16} = \frac{3}{4}$

$\frac{3}{10}+\frac{1}{10} = \frac{4}{10} = \frac{2}{5}$

$\frac{3y}{4y-3}+\frac{7}{4y-3}$

$\frac{3y+7}{4y-3}$

$\frac{5x}{2x+3}+\frac{2}{2x+3}$

$\frac{5x+2}{2x+3}$

$\frac{x}{x-2}+\frac{1}{x-2}$

$\frac{x+1}{x-2}$

$\frac{7x+12}{x+3}+\frac{x^2}{x+3}$

$\frac{x^2+7x+12}{x+3}$

$\frac{(x+3)(x+4)}{x+3}$

$\boxed{x+4}$

$\frac{9x+14}{x+7}+\frac{x^2}{x+7}$

$\frac{x^2+9x+14}{x+7}$

$\frac{(x+2)(x+7)}{x+7}$

$x+2$

$\frac{x^2+8x}{x+5}+\frac{15}{x+5}$

$\frac{x^2+8x+15}{x+5}$

$\frac{(x+3)(x+5)}{x+5}$

$x+3$

Rational Expression Subtraction:
If p, q, and r are polynomials where $r \neq 0$, then

$$\frac{p}{r}-\frac{q}{r}=\frac{p-q}{r}$$

Subtract:

$\frac{n^2}{n-10}-\frac{100}{n-10}$

$\frac{n^2-100}{n-10}$

$\frac{(n-10)(n+10)}{n-10}$

$n+10$

$\frac{4x^2}{2x-5}-\frac{25}{2x-5}$

$\frac{4x^2-25}{2x-5}$

$\frac{(2x-5)(2x+5)}{2x-5}$

$2x+5$

$\frac{y^2}{y-6}-\left(\frac{2y+24}{y-6}\right)$

$\frac{y^2-2y-24}{y-6}$

$\frac{(y-6)(y+4)}{y-6}$ $= y+4$

$\frac{5x^2-7x+3}{x^2-3x-18}-\frac{4x^2+x-9}{x^2-3x-18}$

$\frac{x^2-8x+12}{x^2-3x-18}$

$\frac{(x-6)(x-2)}{(x-6)(x+3)}$

$\frac{x-2}{x+3}$

$\frac{4x^2-11x+8}{x^2-3x+2}-\frac{3x^2+x-3}{x^2-3x+2}$

$\frac{x^2-12x+11}{x^2-3x+2}$

$\frac{(x-11)(x-1)}{(x-2)(x-1)}$

$\frac{x-11}{x-2}$

Preform the indicated operation:

$\frac{4u-1}{3u-1}+\frac{u}{1-3u}$

$\frac{4u-1}{3u-1}+\frac{u}{-(3u-1)}$

$\frac{4u-1}{3u-1}-\frac{u}{3u-1}$

$\frac{3u-1}{3u-1}=1$

$\frac{8x-15}{2x-5}+\frac{2x}{5-2x}$

$\frac{8x-15}{2x-5}+\frac{2x}{-(2x-5)}$

$\frac{8x-15}{2x-5}+\frac{-2x}{2x-5}$

$\frac{6x-15}{2x-5}$

$\frac{3\cancel{(2x-5)}}{\cancel{2x-5}}=3$

$\frac{6y^2+7y-10}{4y-7}+\frac{2y^2+2y+11}{7-4y}$

$\frac{6y^2-7y-10}{4y-7}+\frac{2y^2+2y+11}{-(4y-7)}$

$\frac{6y^2+7y-10}{4y-7}+\frac{-2y^2-2y-11}{4y-7}$

$\frac{4y^2-5y-21}{4y-7}=\frac{\cancel{(4y-7)}(y+3)}{\cancel{4y-7}}$

$y+3$

$\frac{-6}{3\cdot -2}$

$(3n^2+3n)(-2n-2)$

$3n(n+1)$

$\frac{m^2-6m}{m^2-1}-\frac{3m+2}{1-m^2}$

$\frac{m^2-6m}{m^2-1}+\frac{3m+2}{m^2-1}$

$\frac{m^2-3m+2}{m^2-1}$

$\frac{\cancel{(m-1)}(m-2)}{\cancel{(m-1)}(m+1)}$

$\frac{m-2}{m+1}$

$\frac{y^2-5y}{y^2-4}-\frac{6y-6}{4-y^2}$

$\frac{y^2-5y}{y^2-4}+\frac{6y-6}{y^2-4}$

$\frac{y^2+y-6}{y^2-4}$

$\frac{(y+3)\cancel{(y-2)}}{(y+2)\cancel{(y-2)}}$

$\frac{y+3}{y+2}$

$\frac{2n^2+8n-1}{n^2-1}-\frac{n^2-7n-1}{1-n^2}$

$\frac{2n^2+8n-1}{n^2-1}+\frac{n^2-7n-1}{n^2-1}$

$\frac{3n^2+n-2}{n^2-1}$

$\frac{(3n-2)\cancel{(n+1)}}{(n-1)\cancel{(n+1)}}$

$\frac{3n-2}{n-1}$

What you will learn about:
Adding and Subtracting Rational Expressions
With Unlike Demoninators

Find the Least Common Denominator (LCD)
Step 1 – Factor each expression completely

Step 2 – List the factors of each expression. Match the factors vertically when possible

Step 3 – Bring Down the columns.

Find the LCD for $\frac{8}{x^2-2x-3}, \frac{3x}{x^2+4x+3}$

Step 2 (list in columns)

Step 1 (Factor): $x^2 - 2x - 3 = (x+1)(x-3)$

$x^2 + 3x + 4 = (x+1) \qquad (x+3)$

Bring down factors: $(x+1)(x-3)(x+3)$

LCD $= (x+1)(x-3)(x+3)$

Find the LCD for: $\frac{2}{x^2-x-12}, \frac{1}{x^2-16}$

$x^2 - x - 12 \rightarrow (x-4)(x+3)$

$x^2 - 16 \rightarrow (x-4) \qquad (x+4)$

LCD $= (x-4)(x+3)(x+4)$

Find the LCD for $\frac{x}{x^2+8x+15}, \frac{5}{x^2+9x+18}$

$x^2+8x+15 \rightarrow (x+5)(x+3)$

$x^2+9x+18 = (x+3)(x+6)$

LCD $\rightarrow (x+5)(x+3)(x+6)$

Find the LCD for $\frac{-2}{x^2+8x+16}, \frac{4x}{x^2+9x+20}$

$x^2+8x+16 = (x+4)(x+4)$

$x^2+9x+20 = (x+4) \qquad (x+5)$

LCD $(x+4)(x+4)(x+5)$

$(x+4)^2(x+5)$

	Rewrite as equivalent rational expressions with demonimator $(x+1)(x-3)(x+3)$: $\boxed{\frac{8}{x^2-2x-3}}, \boxed{\frac{3x}{x^2+4x+3}}$
Step 1 – Factor each denominator	$x^2-2x-3 = (x+1)(x-3)$ $x^2+4x+3 = (x+1) \quad (x+3)$ $\quad \frac{8}{(x+1)(x-3)} \quad \frac{3x}{(x+3)(x+1)}$ LCD $= \underline{(x+1)(x-3)(x+3)}$
Step 2 - Multiply each denominator by the 'missing' factor and multiply each numerator by the same factor	$\frac{8(x+3)}{(x+1)(x-3)(x+3)}, \frac{3x(x-3)}{(x+1)(x+3)(x-3)}$
Step 3 - Simplify the numerators	$\boxed{\frac{8x+24}{(x+1)(x-3)(x+3)}}, \boxed{\frac{3x^2-9x}{(x+1)(x+3)(x-3)}}$ $\frac{8}{(x+1)(x-3)} \neq \frac{8}{(x+1)(x-3)(x+3)}$
	Rewrite as equivalent rational expressions with demonimator $(x+3)(x-4)(x+4)$: $\frac{2}{x^2-x-12}, \frac{1}{x^2-16}$ $\frac{2\ (x+4)}{(x-4)(x+3)(x+4)} \quad \frac{1\ (x+3)}{(x-4)(x+4)(x+3)}$ $\frac{2x+8}{(x-4)(x+3)(x+4)} \quad \frac{x+3}{(x-4)(x+4)(x+3)}$
	Rewrite as equivalent rational expressions with demonimator $(x+3)(x+5)(x+6)$: $\frac{x}{x^2+8x+15}, \frac{5}{x^2+9x+18}$ $\frac{x^2+6x}{(x+3)(x+5)(x+6)} \quad \frac{5x+25}{(x+3)(x+5)(x+6)}$

Add or Subract

$\frac{7}{12} + \frac{5}{18}$ $\quad$ $\frac{11}{30} - \frac{7}{12}$ $\quad$ $\frac{3}{8} + \frac{9}{20}$

$12 = 2 \cdot 2 \cdot 3$
$21 = 3 \cdot 7$
$2 \cdot 2 \cdot 3 \cdot \boxed{7}$
$84x^2y^2$

$\frac{21}{36} + \frac{10}{36}$ $\quad$ $\frac{22}{60} - \frac{35}{60}$ $\quad$ $\frac{15}{40} + \frac{18}{40}$

$\frac{31}{36}$ $\quad$ $-\frac{13}{60}$ $\quad$ $\frac{33}{40}$

$\frac{5}{12x^2y} + \frac{4}{21xy^2}$ $\quad$ $\frac{2}{15a^2b} - \frac{5}{6ab^2}$ $\quad$ $\frac{5}{16c} - \frac{3}{8cd^2}$

$\frac{35y}{84x^2y^2} + \frac{16x}{84x^2y^2}$ $\quad$ $\frac{4b}{30a^2b^2}$ $\quad$ $\frac{25a}{30a^2b^2}$ $\quad$ $\frac{5d^2}{16cd^2} - \frac{6}{16cd^2}$

$\frac{35y+16x}{84x^2y^2}$ $\quad$ $\frac{4b-25a}{30a^2b^2}$ $\quad$ $\frac{5d^2-6}{16cd^2}$

Add:

$\frac{3}{x-3} + \frac{2}{x-2}$ $\quad$ $\frac{2}{x-2} + \frac{5}{x+3}$ $\quad$ $\frac{4}{m+3} + \frac{3}{m+4}$

Adding Rational Expressions
Step 1 – Determine if the expressions have common denominator.
YES – go to step 2

NO – Rewrite each rational expression with the LCD

Find the LCD

Rewrite each rational expression as an equivalent rational expressions with the LCD

Step 2 – Add the rational expressions

Step 3 – Simplify, if possible

$\frac{2a}{2ab+b^2} + \frac{3a}{4a^2-b^2}$ $\quad$ $\frac{5x}{xy-y^2} + \frac{2x}{x^2-y^2}$ $\quad$ $\frac{7}{2m+6} + \frac{4}{m^2+4m+3}$

$\frac{8}{x^2-2x-3} + \frac{3x}{x^2+4x+3}$ $\quad$ $\frac{1}{m^2-m-2} + \frac{5m}{m^2+3m+2}$