Consider the functions $r(x)$ and $s(x)$, where $r(x) = (x - 3)^2$ and $s(x) = (x + 3)^2$.

What is the degree of $r(x)$? $s(x)$?

$r(x)$ & $s(x)$
Degree $= 2$ & Degree $= 2$

What are the zeros of $r(x)$? $s(x)$?

$r(x)$ & $s(x)$
$\text{zeros}$ & $\text{zeros}$
$x = 3$ & $x = -3$

Consider the function $t(x) = (x - 3)(x + 4)^2$.

a. Expand the expression that defines $t(x)$. Identify the degree of the resulting polynomial.

\begin{align*}
t(x) &= (x - 3)(x^2 + 8x + 16) \\
&= x^3 + 8x^2 + 16x \\
&\quad - 3x^2 - 24x - 48 \\
&= x^3 + 5x^2 - 8x - 48
\end{align*}

b. What are the zeros $t(x)$?

$x = 3, -4$
State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Graph the function on your calculator to verify your answer.

<table>
<thead>
<tr>
<th>Multiplicity Odd</th>
<th>Cross x-axis</th>
<th>Multiplicity Even</th>
<th>Does not cross x-axis</th>
<th>Touch and Turn</th>
</tr>
</thead>
</table>

a) \( f(x) = x(x + 2)^2 \)
   - Degree = 3
   - Zeros: \( x = 0 \) \( 1 \) cross
   - \( x = -2 \) \( 2 \) touch

b) \( f(x) = (x + 3)^3(x - 1)^2 \)
   - Degree = 5
   - Zeros: \( x = -3 \) \( 3 \)
   - \( x = 1 \) \( 2 \) touch

c) \( f(x) = x^3(x - 4) \)
   - Degree = 4
   - Zeros: \( x = 0 \) \( 3 \) cross
   - \( x = 4 \) \( 1 \) cross

Find the polynomial function with leading coefficient 1 that has the given degree and zeros.

A) Degree 3, with 2, -1, and 4 as zeros
   - Zeros: \( x = 2 \) \( x = -1 \) \( x = 4 \)
   - Linear Factor: \( (x - 2)(x + 1)(x - 4) \)
   - \( (x - 2)(x^2 - 3x - 4) \)
   - \( x^3 - 5x^2 + 2x + 8 \)
B) Degree 3 with 5, 1/3, and 2/3 as zeros

\[ x = 5 \quad x = \frac{1}{3} \quad x = \frac{2}{3} \]

Write a polynomial function of minimum degree in factored form with real coefficients whose zeros and their multiplicities include those listed. Then sketch a graph and discuss what you notice.

a) 3 (multiplicity 2), -4 (multiplicity 3)

b) 3 (multiplicity 3), -4 (multiplicity 1)

1 (multiplicity 2), -2 (multiplicity 3)