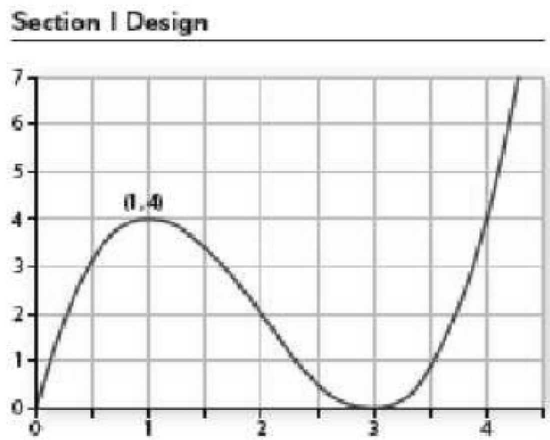
What you will learn about:
Modeling Polynomial Functions

To the n^{th} degree

The following diagram shows the graph of a function that could represent a roller coaster design based on the Section I sketch.



Using the cubic regression tool on your calculator, find a model that will represent the graph.

a. What key points on the graph, besides (1, 4), do you think would be helpful in finding a cubic function that models the proposed Section I Design?

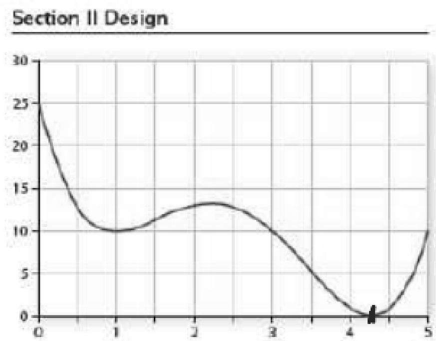
(3,0) (0,0) (4,4)

b. Using the points you selected in Part a, apply a cubic regression routine to find a function for the model.

$y = x^3 - 6x^2 + 9x$

c. Compare the graph of the resulting cubic function to the shape of the Section I Design. Describe ways that the cubic function is or is not a good model of that pattern.

As you have noted, there are some significant differences between the design ideas for Section I and II of the proposed roller coaster. The next diagram shows a graph of a function that could represent a roller coaster design on the Section II sketch.



$y=-x^3+9x^2-23x+25$

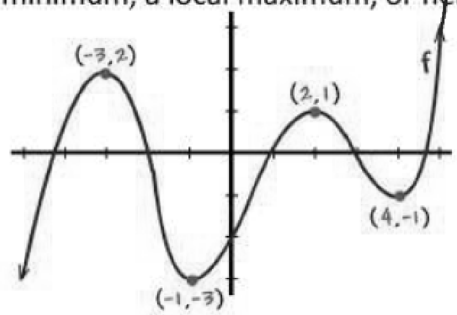
a. Find coordinates of key points outlining the shape of this graph. Then find the cubic function model for the pattern in those points and compare its graph to the shape of the proposed Section II Design.

b. If this is a quartic model, what do you think the minimum number of points needed to find a quadratic model for a data or graph pattern?

c. Find a quartic function model for the pattern of data points you identified in Part a. Check how well its graph matches the pattern in the Section II Design.

Standard Form of a Polynomial	
$y = ax^n + bx^{n-1} + cx^{n-2} + \ldots$	Determine which are Polynomial Functions. For those that are, state the degree and leading coefficient.
Degree of a polynomial Highest power of variable.	a) $f(x) = 4x^3 - 5x - .5$ $4x^3 - 5x - \frac{1}{2}$ Yes polynomial function Degree 3 L.C. 4 b) $g(x) = 6x^{4} + 7$ Not a polynomial Function
Leading Coefficient # in front of variable that gives the degree.	c) $h(x) = \sqrt{9x^4 + 16x^2}$ $(9x^4 + 16x^2)^{1/2}$ Not a polynomial d) $k(x) = 15 - 2x^4$ $= -2x^4 + 15$ Yes polynomial function Deg = 4 L.C. = -2
	e) $f(x) = 2x^2 + 5x - 12$ f) $f(x) = x^3 - x^2 - 6x$
monomial $x,\ 3x,\ 5,\ -37{,}000$ $x^2y^5z^{10}$ binomial $x+3$	g) $f(x) = x^3 - 25x$ h) $r(x) = 3x^0$ Deg = 0 L.C. = 3

State whether each labeled point identifies a local minimum, a local maximum, or neither.



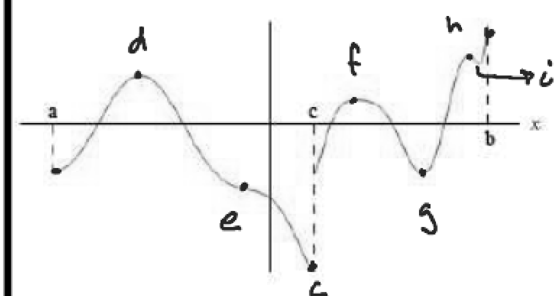
(-3,2) → Local max

(-1,3) → Local min

(2,1) → Local max

(4,-1) → Local min

Local max	Local min	Neither
d	a	e
f	c	
h	g	
b	i	



Using your calculator, graph each function and give the coordinates of any local minimum(s) and local maximum(s).

$g(x) = -x^3 + 6x^2 - 9x$

Local min (1,-4)

Local max (3,0)

$h(x) = -x^4 + 2x^3 + 7x^2 - 8$

$h(x) = -x^3 + 2x - 3$

$\cancel{f(x) = \sqrt[3]{x+4}}$