PRACTICE QUIZ: UNIT 5 – LESSON 1

1. Consider the graph of the polynomial function shown below.

a. What is the smallest possible degree for this polynomial function? Explain your reasoning.

   4th Degree 4 Zeros
   x-intercepts 3 Local Extrema (Turning points)

b. Estimate the coordinates of all local maximum points.
   \((-3.8, 8)\) \((2.9, 8)\)

c. Estimate the coordinates of all local minimum points.
   \((-5, -4.25)\)

d. Estimate the zeroes of this polynomial function.
   \(x = -5, -1, 2, 4\)

e. Create a Rule for this polynomial.
   \(f(x) = -0.1x^4 - 2x^3 + 2.1x^2 + 2.2x - 4\)

f. Over what intervals is the function increasing?
   \((-\infty, -3.8)\) \((-5, 2.9)\)

g. Over what intervals is the function decreasing?
   \((-3.8, -5)\) \((2.9, \infty)\)
2. Consider the function $f(x) = (x^3 - x)(x + 5)$.
   a. What are the zeroes of $f(x)$?
   
   $x = 0, 1, -5$

   b. Write $f(x)$ in standard polynomial form.
   
   $(x^2 - x)(x + 5)$
   
   $x^3 + 5x^2 - x^2 - 5x$
   
   $f(x) = x^2 + 4x^2 - 5x$

3. Write a rule for a cubic function that has zeroes only at (3, 0) and (-2, 0). Explain your reasoning.

   $f(x) = (x-3)(x+2)^2$ or $f(x) = (x-3)(x+2)$

   One of zeros has to have a multiplicity of 2.

4. The daily revenue of the Lensic movie theater depends on the ticket cost $x$ according to the function $R(x) = x(250 - 5x)$. The daily expenses of the theater are related to the ticket price according to the rule $E(x) = 1250 - 30x$.
   a. Find a rule in standard polynomial form for the daily profit.

   $P(x) = R(x) - E(x)$
   
   $= (250x - 5x^2) - (1250 - 30x)$
   
   $= 250x - 5x^2 - 1250 + 30x$
   
   $= 280x - 5x^2 - 1250$
   
   b. If the ticket price is $8$, what is the daily profit?

   $P(8) = -5(8)^2 + 280(8) - 1250$
   
   $= 670$

Simplify the following expressions. Show your work where applicable.

1. $(x+2y)^2$

2. $(x+1)(3x^2 + 2x - 1)$

3. $(x-1)/(3x + 2)(x-4)$

4. $(x^3+1)-(4x^2+5)+(x^2-x-2)$

5. $6(x^3+4x^2-3)-4(2x^3-3x)$
(2x+3)(2x+5)(2x+3) 7. (7x^2 - 6x - 8)(2x + 2)

(2x+3)(4x^2 + 12x + 9) 8. (x+4)(3x + 2)(x - 4)

8x^3 + 24x^2 + 18x 14x^2 - 12x + 16

-14x^3 + 12x^2 + 16x -14x^3 + 26x^2 + 4x - 16

\[ \frac{12x^2 + 36x + 27}{8x^3 + 36x^2 + 54x + 27} \]

9. (2x^2 + 5)^2 10. (15x^2 - 9x + 9) - (13x^2 + 15x + 5) - (-16x^2 + 20x + 16) + (-7x^2 + 10x - 10)

(2x^2+5)(2x+5) 18x^2 - 2x + 9 - 13x^2 = 18x - 5 + 16x^2 - 20x - 16 - 7x^2 + 10x - 10

4x^4 + 10x^2 + 10x^2 + 25 11x^2 - 34x - 22

4x^4 + 20x^2 + 25

11. (-7x^2 + 14x + 17) + (19x^2 - 6x - 20x^2) - (-14x^4 - 18x^2 + 20) + (20x^2 + 19x + 17 - 3x^2)

-7x^2 + 14x + 17 + 19x^2 - 6x - 20x^2 + 14x^4 + 18x^2 - 20 + 20x^2 + 19x + 17 - 3x^2

11x^6 - 20x^5 + 37x^4 + 32x^2 + 8x + 14

Give the degree of the polynomial. List the zeros, give the multiplicity and tell whether the graph crosses or touches the x-axis.

a) \( f(x) = x^3(x-4)(x+1)^2 \) D = 6  
   \begin{align*}
   \text{Zeros} & \quad \text{mult} \\
   x = 0 & \quad 2 \quad \text{touch} \\
   x = 1 & \quad 3 \quad \text{Cross} \\
   x = -1 & \quad 3 \quad \text{Cross} \\
   \end{align*}

b) \( f(x) = (x-4)(x+1)^2 \) D = 3  
   \begin{align*}
   \text{Zeros} & \quad \text{mult} \\
   x = 4 & \quad 1 \quad \text{Cross} \\
   x = 0 & \quad 1 \quad \text{Cross} \\
   \end{align*}

c) \( f(x) = -x(x+3)^2(x-1)^2 \) D = 5  
   \begin{align*}
   \text{Zeros} & \quad \text{mult} \\
   x = 0 & \quad 1 \quad \text{Cross} \\
   x = -3 & \quad 2 \quad \text{touch} \\
   x = 1 & \quad 2 \quad \text{touch} \\
   \end{align*}

John runs two movie theater companies. He has decided that admission prices at the two movie theaters should be the same. The weekly profit (in dollars) at each of the company is given by the following function rules, where \( t \) is the ticket price.

Raging Waters \( P_r(t) = -5t^2 + 150t - 500 \)  
Wonderful Waves \( P_w(t) = -5t^2 + 200t - 1000 \)

a. Write a function rule for the combined weekly profit of the two water parks.

\[
P_r(t) + P_w(t) = -10t^2 + 350t - 1500
\]

b. What weekly profit will John make if the ticket price is $22?

\[
-10(22)^2 + 350(22) - 1500
\]

\$1360
Suppose a new animated movie will feature a main character that is a two-humped camel. The graph below indicates the shape of the camel’s humps.

![Graph of a two-humped camel shape](image)

a. Explain why it is reasonable to use a fourth-degree (quartic) polynomial to model this curve.

4 x-intercepts or 3 local extrema

b. When trying to find a polynomial function rule that would closely match the graph, Dawn began with $f(x) = (x - 1)(x - 5)(x + 3)(x + 1)$.

i. Explain why this function rule is a reasonable first choice.

Same x-intercepts

ii. How do the shape, zeroes, and end behavior of the graph of $f(x)$ compare to those of the graph above? Use the window $x[-4, 6]$ and $y[-100, 50]$.

Shape - Reflected over x-axis
Zeroes same
End behaviors are opposite

iii. Provide a reasonable set of control points that could be used to find a rule that matches the graph above. Use those points and your calculator to find the regression model that best fits.

$(1, 0)$ $(5, 0)$ $(-3, 0)$ $(-1, 0)$ $(0, -18)$

$f(x) = -1.2x^4 + 2.4x^2 + 19.2x^2 - 2.4x - 18$
Consider the polynomial \( p(x) = x^2 + 2x^2 - 6x - 9 \)

a. Use long division or synthetic division to determine if \((x + 3)\) is a factor of \(p(x)\)?

\[
\begin{array}{c|ccccc}
-3 & 1 & 2 & -6 & -9 \\
-3 & & 3 & 9 & \\
\hline
1 & -1 & -3 & 0
\end{array}
\]

Yes, \((x + 3)\) is a factor.

b. Use long division to divide \(p(x)\) by the linear term \((x - 2)\) and express the result as an equation in the form

\[
p(x) = (x - 2)(x^3 + 4x^2 - 6x - 9)\]

\[
\begin{array}{c|cccc}
\text{(-)} & 1 & 4 & -6 & -9 \\
\text{(-)} & x & 4x & -8x & \\
\hline
\text{(-)} & x^3 & 2x^2 & 0 & \frac{5}{2}x = 2
\end{array}
\]

Write each expression as a product of linear factors and then find the zero's.

a. \(8x^2 - 10x + 3 = 0\)

\[
(4x - 3)(2x - 1) = 0
\]

\[
x = \frac{3}{4}, \quad x = \frac{1}{2}
\]

b. \(3x^2 + 5x - 2 = 0\)

\[
(3x - 1)(x + 2) = 0
\]

\[
x = \frac{1}{3}, \quad x = -2
\]

Using Long division and algebra find the zeroes and the linear factors for the function below.

\(f(x) = 5x^3 + 21x^2 - 21x - 5\)  \(\text{factor: } x + 5\)

\[
\begin{array}{c|cccc}
-5 & 5 & 21 & -21 & -5 \\
\hline
& -25 & 20 & 5 & \\
\hline
& 5 & -4 & -1 & 0
\end{array}
\]

\(5x^2 - 4x - 1 = 0\)

\[
(5x + 1)(x - 1) = 0
\]

\[
x = -\frac{1}{5}, \quad x = 1
\]

Zeros:

\[
x = -5, \quad -\frac{1}{5}, \quad 1
\]
Write a polynomial function in standard form of least degree with integral coefficients that has the given zeros.

\[ x = \frac{1}{2} \]
\[ 2 \times \frac{1}{2} = \frac{1}{2} x - 1 = 0 \]

a. \(3, -5, \frac{1}{2}\)
\[(x-3)(x+5)(2x-1)\]
\[(x^2+2x-15)(2x-1)\]
\[2x^3+4x^2-30x\]
\[-x^2-2x+15\]
\[\frac{2x^5+3x^4-32x^3+15x}{2x^3+3x^2-32x+15}\]

b. \(-2, -4, 1\)
\[(x+2)(x+4)(x-1)\]
\[(x+2)(x^2+3x-4)\]
\[x^3+3x^2-4x\]
\[2x^2+6x-8\]
\[x^3+5x^2+2x-8\]

Write a polynomial function of minimum degree in factored form with real coefficients whose zeros and their multiplicities include those listed.

a. \(-1\) (multiplicity 3), \(2\) (multiplicity 2), \(0\) (multiplicity 1)
\[ f(x) = x(x+1)^3(x-2)^2 \]

b. \(5\) (multiplicity 1), \(-7\) (multiplicity 3)
\[ f(x) = (x-5)(x+7)^3 \]
Determine the end behavior.

a. \( f(x) = 5x^3 + 9x^2 - 26x - 24 \)

\[
\lim_{x \to \infty} f(x) = -\infty
\]

\[
\lim_{x \to -\infty} f(x) = \infty
\]

d. \( f(x) = -2(x-1)(x-4)(x-6)(x-9) \)

\[
\lim_{x \to \infty} f(x) = -\infty
\]

\[
\lim_{x \to -\infty} f(x) = -\infty
\]

c. \( \lim_{x \to \infty} f(x) = -\infty \)

\[
\lim_{x \to -\infty} f(x) = \infty
\]