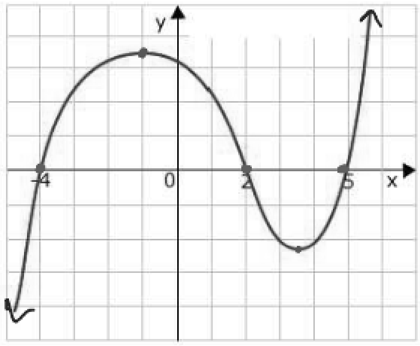
Name:______________________________ Block:__________

PRACTICE TEST: UNIT 5 – POLYNOMIAL AND RATIONAL FUNCTIONS



1) Consider the graph of the polynomial function shown below.

a. What is the smallest possible degree for this polynomial function? Explain your reasoning.

Degree = 3

3 x-intercepts

2 turning point

b. Estimate the coordinates of all local maximum points.

(-1, 3.5)

c. Estimate the coordinates of all local minimum points.

(3.5, -2.25)

d. Estimate the zeroes of this polynomial function.

x = -4, 2, 5)

e. Use the regression capabilities of your calculator to find a polynomial function rule that models the graph pattern. List the points you use as the basis of your regression.

Points: (-4,0) (2,0) (5,0) (-1, 3.5)

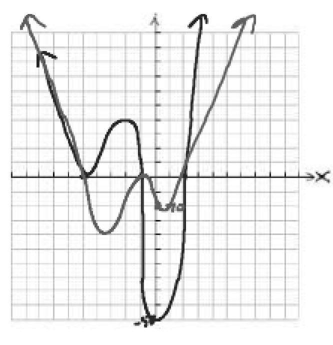
Function Rule: $.06x^3 - .19x^2 - 1.17x + 2.59$

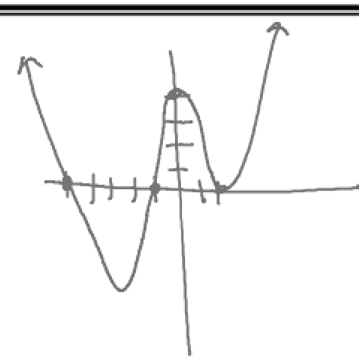
3) Write a rule for a quartic function that has zeros only at (-5, 0), (-1, 0), (2, 0). Then sketch the graph that matches your rule, including a scale.

$y = (x+5)^2(x+1)(x-2)$
y-intercept (0,-50)

$y = (x+5)(x+1)^2(x-2)$
y-intercept (0,-10)

$y = (x+5)(x+1)(x-2)^2$
y-intercept (0,20)





4) Consider the graph of the rational function $f(x)=\frac{2x^2-18}{x^2-4}$. $= \frac{2(x^2-9)}{x^2-4} = \frac{2(x+3)(x-3)}{(x+2)(x-2)}$

a. Describe the domain of $f(x)$.

D: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

P.O.D $x=\pm 2$

b. What are the coordinates of all x-intercepts of the graph? Explain how they are related to the function rule for $f(x)$.

$2x^2-18=0$

$2(x-3)(x+3)=0$

$x=3,\ x=-3$

they are x-values that make only the top zero.

c. Find equation(s) of all vertical asymptotes of $f(x)$. Explain how the vertical asymptotes are related to the function rule for $f(x)$.

$x^2-4=0$

$(x+2)(x-2)=0$

$x=-2\ \ x=2$

They are x-values that only make the bottom zero.

d. Find the equation of the horizontal asymptotes of $f(x)$. Explain how the horizontal asymptotes are related to the function rule for $f(x)$.

$y=2 \qquad y=\frac{2}{1}$

If Degree ontop equals degree on Bottom then H.A. is the ratio of Leading coefficents

6. Consider the following expressions. (A) Give the domain. (B) Simplify the expression. (C) Identify what values of x (if any) are the original expression and simplified expression not equivalent (Find the hole(s)). Show all work!

a. $\frac{x^2-81}{x^2+13x+36}$

$\frac{(\cancel{x+9})(x-9)}{(\cancel{x+9})(x+4)}$

A) D: $(-\infty,-9)\cup(-9,-4)\cup(-4,\infty)$

B) $\frac{x-9}{x+4}$

c) Hole $x=-9$

P.O.D $x=-\frac{2}{3},-6$

b. $\frac{4x^2+24x}{3x^2+20x+12}$

$\frac{4x(\cancel{x+6})}{(3x+2)(\cancel{x+6})}$

A) $(-\infty,-6)\cup(-6,-\frac{2}{3})$ $(-\frac{2}{3},\infty)$

B) $\frac{4x}{3x+2}$

C) Hole $x=-6$

P.O.D $x=\pm 12$

c. $\frac{n^2+15n+36}{n^2-144}$

$\frac{(\cancel{n+12})(n+3)}{(\cancel{n+12})(n-12)}$

A) $(-\infty,-12)\cup(-12,12)\cup(12,\infty)$

B) $\frac{n+3}{n-12}$

C) Hole $x=-12$

P.O.D $x=-8,3$

d. $\frac{x-6}{x^2+5x-24}$

$(x+8)(x-3)$

A) $(-\infty,-8)\cup(-8,3)\cup(3,\infty)$

B) $\frac{x-6}{(x+8)(x-3)}$

C) None

7. Does the graph of the function $\frac{x^2-81}{x^2+13x+36}$ have 0, 1, or 2 vertical asymptotes? Explain how you could determine this without looking at a graph of the function.

$$\frac{\cancel{(x+9)}(x-9)}{\cancel{(x+9)}(x+4)}$$

1 → V.A → (only one x-value that just makes bottom zero)

The other P.O.D. is a hole

8. Give everything about the following functions (*Hint*: zeros, asymptotes, intercepts, holes, domain) and sketch a graph. **Draw your axis and label your scale**

I. $y=\frac{x+4}{x^2-36}$ $(x-6)(x+6)$ $x+4=0$, $x=-4$

Zeros: $x=-4$

Y-intercepts: $\left(0,\frac{4}{-36}\right)=\left(0,-\frac{1}{9}\right)$

Points of Discontinuity: $x=\pm 6$

Holes: None

Vertical Asymptotes: $x=\pm 6$

Horizontal Asymptotes: $y=0$

Domain: $(-\infty,-6)\cup(-6,6)\cup(6,\infty)$

II. $f(x)=\frac{x}{x^2-2x-3}$ $(x-3)(x+1)$

Zeros: 0

Y-intercepts: $(0,0)$

Points of Discontinuity: $x=-1, 3$

Holes: None

Vertical Asymptotes: $x=-1,\ 3$

Horizontal Asymptotes: $y=0$

Domain: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$

III. $g(x)=\frac{x^2-16}{x^2-1}$ $\frac{(x+4)(x-4)}{(x+1)(x-1)}$

Zeros: $x=\pm 4$

Y-intercepts: $(0,16)$

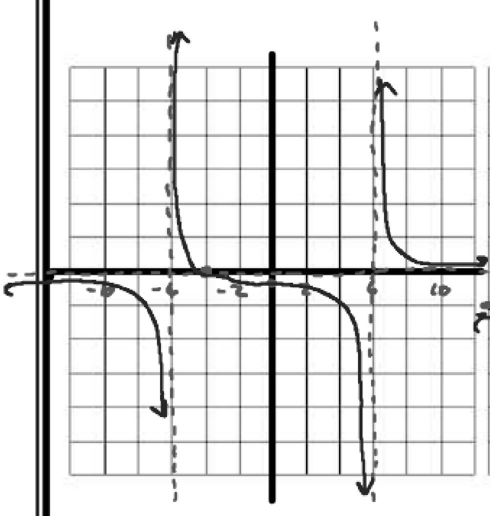
Points of Discontinuity: $x=\pm 1$

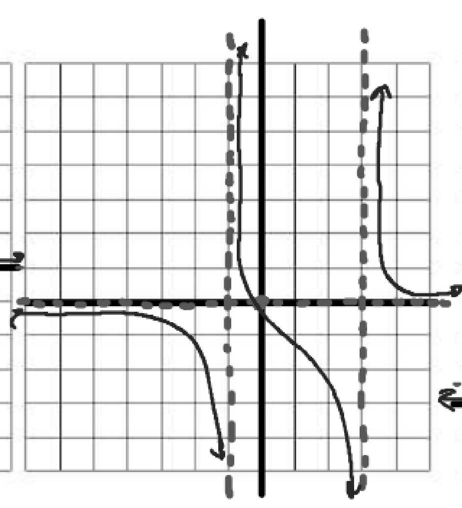
Holes: None

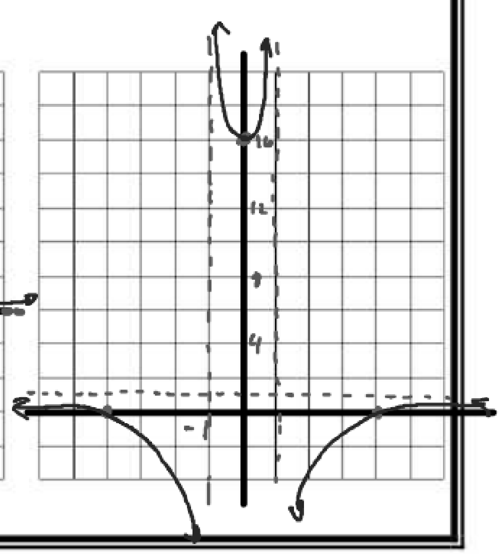
Vertical Asymptotes: $x=\pm 1$

Horizontal Asymptotes: $y=1$

Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$







9. Write each sum or difference in a simplified form.

a. $\frac{2x+5}{x-1}+\frac{x-6}{x-1}$

$\frac{3x-1}{x-1}$

b. $\frac{15}{3x+5}-\frac{5-6x}{3x+5}$

$\frac{10+6x}{3x+5}$

$\frac{6x+10}{3x+5}=\frac{2(3x+5)}{3x+5}=2$

c. $\frac{x^2+2}{x(x+4)}+\frac{9(x+4)}{x(x+4)}$

$\frac{x^3+2x}{x(x+4)}+\frac{9x+36}{x(x+4)}$

$\frac{x^3+11x+36}{x(x+4)}$

d) $\frac{2y-5}{y^2-25}-\frac{5}{y^2-25}$

$\frac{2y-10}{y^2-25}$

$\frac{2(y-5)}{(y-5)(y+5)}=\frac{2}{y+5}$

e) $\frac{9x}{3x+5}+\frac{15}{3x+5}$

$\frac{9x+15}{3x+5}$

$\frac{3(3x+5)}{3x+5}=3$

f) $\frac{k(k+2)}{k-2}-\frac{8}{k^2-4}$ $(k+2)(k-2)$

$\frac{k^2+2k}{(k+2)(k-2)}-\frac{8}{(k+2)(k-2)}$

$\frac{k^2+2k-8}{(k+2)(k-2)}=\frac{(k+4)(k-2)}{(k+2)(k-2)}$

$\frac{k+4}{k+2}$

g) $\frac{3(x+1)}{x}+\frac{x-3}{3x}$

$\frac{3x+3}{3x}+\frac{x-3}{3x}$

$\frac{4x}{3x}=\frac{4}{3}$

h) $\frac{1(y-6)}{y+6}+\frac{1(y+6)}{y-6}+\frac{12}{y^2-36}$ $(y-6)(y+6)$

$\frac{y-6}{(y-6)(y+6)}+\frac{y+6}{(y-6)(y+6)}+\frac{12}{(y-6)(y+6)}$

$\frac{2y+12}{(y+6)(y-6)}=\frac{2(y+6)}{(y+6)(y-6)}=\frac{2}{y-6}$

i) $\frac{10y}{y^2-25}+\frac{5(y+5)}{(y-5)(y+5)}$ $(y-5)(y+5)$

$\frac{10y}{(y-5)(y+5)}+\frac{5y+25}{(y-5)(y+5)}$

$\frac{15y+25}{(y-5)(y+5)}$

$\frac{5(3y+5)}{(y-5)(y+5)}$

10. Rewrite each product or quotient in an equivalent simpler form.

a. $\frac{x}{x^2+5x}\div\frac{x}{x^2-25}$

$\frac{x}{x(x+5)}\cdot\frac{(x+5)(x-5)}{x}$

$\frac{x-5}{x}$

b. $\frac{x^2+7x+12}{4x+12}\cdot\frac{x+5}{x+4}$

$\frac{(x+4)(x+3)}{4(x+3)}\cdot\frac{x+5}{x+4}$

$\frac{x+5}{4}$

c) $\frac{y^2-4y}{y^2+3y}\cdot\frac{y^2-9}{y-4}$

$\frac{y(y-4)}{y(y+3)}\cdot\frac{(y+3)(y-3)}{y-4}$

$y-3$

d) $\frac{2y}{y-7}\cdot\frac{7-y}{14}$

$y-7\neq 7-y$

$7-y$

$-y+7$

$-(y-7)$

$\frac{2y}{y-7}\cdot\frac{-1(y-7)}{14}$

$\frac{-2y}{14}$ 7

e) $\frac{4x-20}{4x+20}\cdot\frac{3x^2+30x}{3x^2-15x}$

$\frac{4(x-5)}{4(x+5)}\cdot\frac{3x(x+10)}{3x(x-5)}$

$\frac{x+10}{x+5}$

f) $\frac{x^2-4x-32}{x^2+12x+32}\div\frac{(x-8)^2}{x^2-64}$

$\frac{(x-8)(x+4)}{(x+8)(x+4)}\cdot\frac{(x-8)(x+8)}{(x-8)(x-8)}$

1

$\frac{-y}{7}$

$9-2x \neq 2x-9$

g) $\dfrac{(y+1)^2}{y^2+y} \div \dfrac{y^2-1}{y^2}$

$\dfrac{(\cancel{y+1})(\cancel{y+1})}{Y(\cancel{y+1})} \cdot \dfrac{Y^2}{(y-1)(\cancel{y+1})}$

$\dfrac{Y}{Y-1}$

h) $\dfrac{2x-9}{x+1} \div \dfrac{9-2x}{3x+3}$

$9-2x$

$-2x+9$

$-(2x-9)$

$\dfrac{\cancel{2x-9}}{\cancel{x+1}} \cdot \dfrac{3(\cancel{x+1})}{-(\cancel{2x-9})}$

$\dfrac{3}{-1} = -3$

i) $\dfrac{b+8}{5b^2} \cdot \dfrac{3b^2-24b}{b^2-64}$

$\dfrac{\cancel{b+8}}{5b^{\cancel{2}}} \cdot \dfrac{3\cancel{b}(\cancel{b-8})}{(\cancel{b+8})(\cancel{b-8})}$

$\dfrac{3}{5b}$

Perform the operation and simplify

1. $\dfrac{2x^2-10x}{x^2-25} \bullet \dfrac{x+3}{2x^2}$

$\dfrac{\cancel{2x}(\cancel{x-5})}{(x+5)(\cancel{x-5})} \cdot \dfrac{x+3}{\cancel{2}x^{\cancel{2}}}$

$\dfrac{x+3}{x(x+5)}$

3. $\dfrac{4x}{5x-20} \div \dfrac{x^2-2x}{x^2-6x+8}$

$\dfrac{4\cancel{x}}{5(\cancel{x-4})} \cdot \dfrac{(\cancel{x-4})(\cancel{x-2})}{\cancel{x}(\cancel{x-2})}$

$\dfrac{4}{5}$

4. $\dfrac{2x^2+3x-5}{6x} \div \dfrac{(2x^2+5x)}{1}$

$\dfrac{(\cancel{2x+5})(x-1)}{6x} \cdot \dfrac{1}{x(\cancel{2x+5})}$

$\dfrac{x-1}{6x^2}$

5. $\dfrac{1}{3x^2}(3x-4) + \dfrac{x}{9x^2-12x}(x)$

$3x(3x-4)$

$\dfrac{3x-4}{3x^2(3x-4)} + \dfrac{x^2}{3x^2(3x-4)}$

$\dfrac{x^2+3x-4}{3x^2(3x-4)} = \dfrac{(x+4)(x-1)}{3x^2(3x-4)}$

6. $\dfrac{12(x)}{x^2-x-12} + \dfrac{5(x+3)}{12x-48}$

$(x-4)(x+3) \quad 12(x-4)$

$\dfrac{12x}{12(x-4)(x+3)} + \dfrac{5x+15}{12(x-4)(x+3)}$

$\dfrac{17x+15}{12(x-4)(x+3)}$

7. $\dfrac{(x+1)(x-2)}{x^2+4x+4} - \dfrac{6(x+2)}{x^2-4}$

$(x+2)(x+2) \quad (x-2)(x+2)$

$\dfrac{x^2-x-2}{(x+2)(x+2)(x-2)} - \dfrac{6x+12}{(x+2)(x+2)(x-2)}$

$\dfrac{x^2-7x-14}{(x+2)(x+2)(x-2)}$