


What you will learn about:
Modeling Polynomial Functions

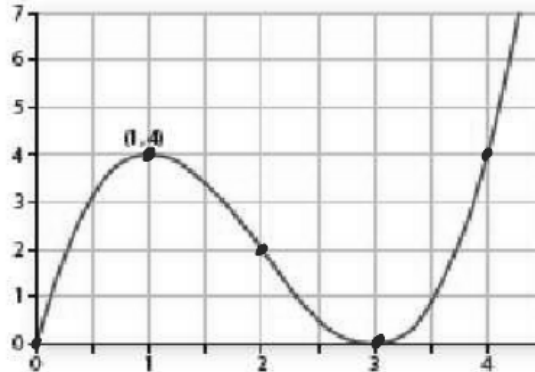
$$f(x) = ax^n + bx^{n-1} + cx^{n-2} + \dots$$

$$x^2 + 7x + 4$$


To the n^{th} degree

The following diagram shows the graph of a function that could represent a roller coaster design based on the Section I sketch.

Section I Design



Using the cubic regression tool on your calculator, find a model that will represent the graph.

- a. What key points on the graph, besides (1, 4), do you think would be helpful in finding a cubic function that models the proposed Section I Design?

$$(0,0) \quad (1,4) \quad (2,2) \quad (3,0) \quad (4,4)$$

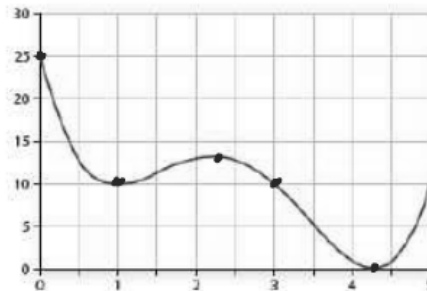
- b. Using the points you selected in Part a, apply a cubic regression routine to find a function for the model.

$$y = x^3 - 6x^2 + 9x$$

- c. Compare the graph of the resulting cubic function to the shape of the Section I Design. Describe ways that the cubic function is or is not a good model of that pattern.

As you have noted, there are some significant differences between the design ideas for Section I and II of the proposed roller coaster. The next diagram shows a graph of a function that could represent a roller coaster design on the Section II sketch.

Section II Design



- a. Find coordinates of key points outlining the shape of this graph. Then find the cubic function model for the pattern in those points and compare its graph to the shape of the proposed Section II Design.

$$(1, 10) \quad (3, 10) \quad (5, 10) \quad (0, 25)$$

$$y = -x^3 + 9x^2 - 23x + 25$$

- b. If this is a quartic model, what do you think the minimum number of points needed to find a quadratic model for a data or graph pattern?

$$y = -1.06x^4 - 10.62x^3 + 37.58x^2 - 39.03x + 25$$

min # of points 5 pts.

- c. Find a quartic function model for the pattern of data points you identified in Part a. Check how well its graph matches the pattern in the Section II Design.

Standard Form of a Polynomial

Start with highest power and work down

Degree of a polynomial

Highest power of Variable

Leading Coefficient

Determine which are Polynomial Functions. For those that are, state the degree and leading coefficient.

a) $f(x) = 4x^3 - 5x - .5$

$$4x^3 - 5x - \frac{1}{2}$$

Yes polynomial Function

Degree = 3

$$L.C. = 4$$

pad math
↓

c) $h(x) = \sqrt{9x^4 + 16x^2} = 3x^2 + 4x$

No not polynomial Function

e) $f(x) = 2x^2 + 5x - 12$

Yes Polynomial Function

Degree: 2

$$L.C. = 2$$

g) $f(x) = x^3 - 25x$

b) $g(x) = 6x^{-4} + 7$

$$\frac{6}{x^4} + 7$$

Not Polynomial Function

d) $k(x) = 15 - 2x^4 - 2x^4 + 5$

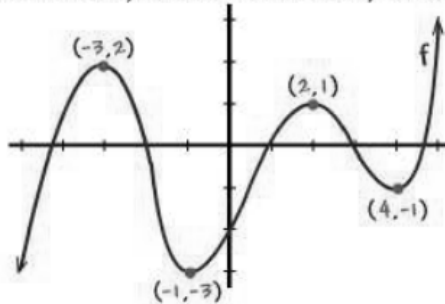
Yes Polynomial Function

$$D = 4$$

$$L.C. = -2$$

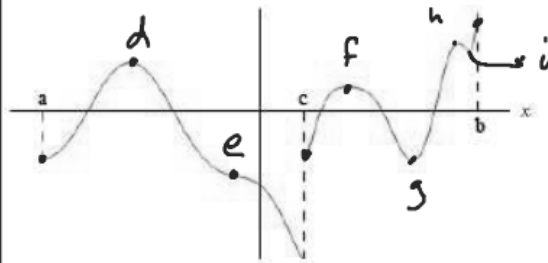
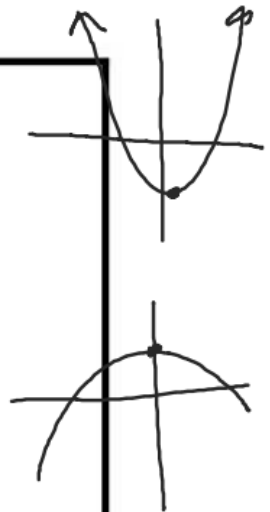
f) $f(x) = x^3 - x^2 - 6x$

State whether each labeled point identifies a local minimum, a local maximum, or neither.



Local max $(-3, 2), (2, 1)$

Local min $(-1, -3), (4, -1)$



- a) Local min
- b) Local max
- c) Local min
- d) Local max
- e) Neither
- f) Local max
- g) Local min
- h) Local max
- i) Local min

Using your calculator, graph each function and give the coordinates of any local minimum(s) and local maximum(s).

$$g(x) = -x^3 + 6x^2 - 9x$$

Local min $(1, -4)$

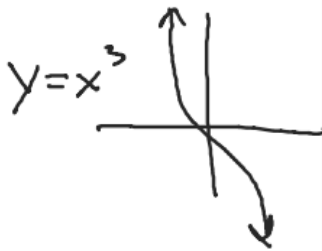
Local max $(3, 0)$

$$h(x) = -x^4 + 2x^3 + 7x^2 - 8$$

$$h(x) = -x^3 + 2x - 3$$

~~$$f(x) = x^2 \sqrt{x+4}$$~~

Why are the points $(0,0)$ and $(4.25, 7)$ not considered local minimum or local maximum points for the cubic function you found in problem 2 to model the Section I design for the roller coaster.



Consider the relationship between the degree of a function and the number of local maximum and/or local minimum points on the graph of a function.

- a. Give an example of a polynomial function with no local maximum or local minimum point.

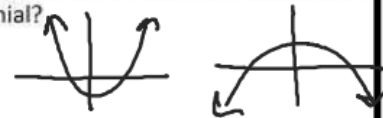
Linear equation

$$y = 2x + 3$$



- b. How many local maximum and/or minimum points can there be on the graph of a quadratic polynomial?

1 Max/min



Given the equation, $y = x^3 - 6x^2 + 12x - 3$, how many local max/min would you expect to have? Find them.

Given the equation, $y = x^4 - 9x^2 + 2$, how many local max/min would you expect to have? Find them.

Given the equation, $y = x^5 - 5x^3 + 4x$, how many local max/min would you expect to have? Find them.