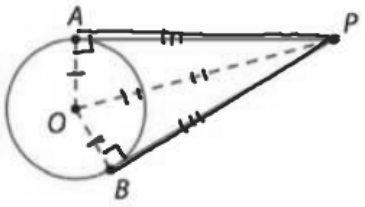
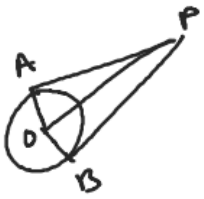
Now suppose you are given that $\overline{PA}$ *and* $\overline{PB}$ are tangents to a circle centered at O. To help prove that $\overline{PA} \cong \overline{PB}$, auxiliary line segments $\overline{OA}, \overline{OB}, and\ \overline{OP}$ are drawn in the figure.



CPCTC

How could you use congruent triangles to prove $\overline{PA} \cong \overline{PB}$?



How could you use the Pythagorean Theorem to show that $\overline{PA} \cong \overline{PB}$?

$OA^2 + AP^2 = OP^2$ $\qquad$ $OB^2 + BP^2 = OP^2$

$OA^2 + AP^2 = OB^2 + BP^2$

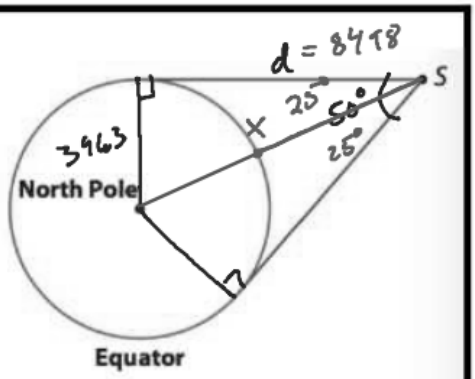
$AP^2 = BP^2$

$AP = BP$

State in words the theorem you have proved about tangents drawn to a circle from an exterior point.

Tangents to a circle from the same external pt are $\cong$.

Suppose a satellite is located in space at point S. In this view of Earth in the plane of the equator, the angle between the lines of sight at S is 50°. The radius of the Earth is 3,963 miles.



What is the distance from S to the horizontal along the equator, that is, the length of a tangent from S to the Earth's surface along the equator?

$$d(\tan 25°) = \left(\frac{3963}{d}\right)d$$

$$\frac{d\tan 25}{\tan 25} = \frac{3963}{\tan 25}$$

$$d = \frac{3963}{\tan 25} = 8498 \text{ mi}$$

How high is the satellite S above Earth's surface, that is, the length of a segment S to the closest point on Earth's surface along the equator?

$$3963^2 + 8498^2 = c^2$$

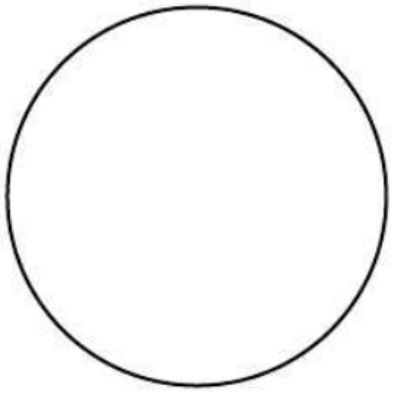
$$c = 9368.48$$

$$9368.48 - 3963$$

$$5405.48$$

Chords, Arcs, and Central Angles

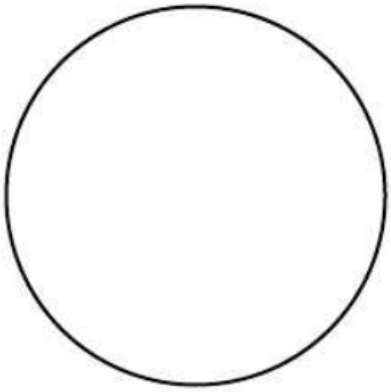
Chords



Central Angle

Minor Arc

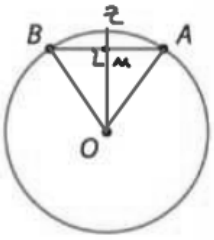
Major Arc



Congruent Arcs

Relationships between Chords

Given the figure at the right.



Estimate the midpoint M on segment $\overline{AB}$ and label that point.

Draw a line through O and M so that $\overline{OM} \perp \overline{AB}$.

What Three things happen?

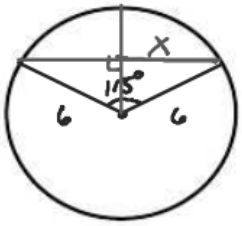
- Radius Bisects Chord $\overline{AM} \cong \overline{BM}$
- Radius Bisects Intercepted Arc $\overset{\frown}{AZ} \cong \overset{\frown}{BZ}$
- Radius Bisects Central Angle $\angle AOM \cong \angle BOM$

Congruent Chords

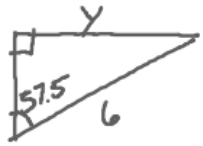
Same distance from center

Suppose that a given circle has a radius of 6 inches.

What is the length of a chord that has a central angle of 115°?



Chord 10.12 in



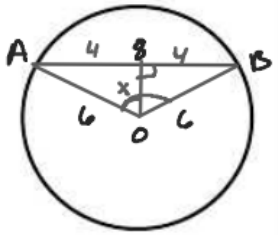
$6(\sin 57.5) = \left(\frac{Y}{6}\right)6$

$6 \sin 57.5 = Y$

$5.06 = Y$

SOH-CAH-TOA

What is the measure of the arc of a chord that is 8 inches long? What is the perpendicular distance from the center of the circle to the chord?

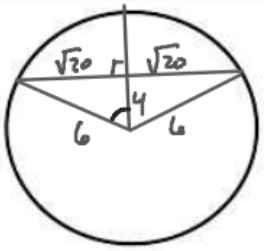


$m\overset{\frown}{AB} = 84°$

$x = \sqrt{20}$

$\cos^{-1}\left(\frac{4}{6}\right) = 48.19$

The perpendicular distance from the center of the circle to a chord is 4 inches. What is the length of the chord? What is the measure of its central angle?

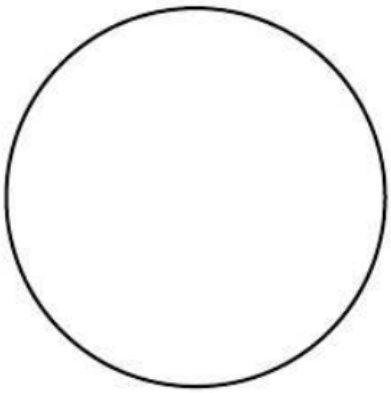


Chord $= 2\sqrt{20}$

Central $\angle = 96^\circ$

Inscribed Angles in Circles

Inscribed Angles



Congruent Inscribed Angles