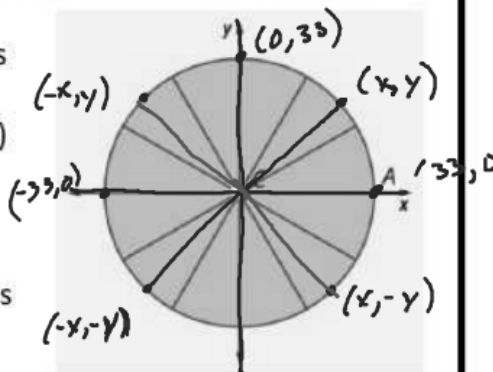


What you will learn about:  
Modeling Circular Motion

The Ferris wheel was invented in 1893 as an attraction at the World Columbian Exhibition in Chicago, and it remains a popular ride at carnivals and amusement parks around the world.

### COORDINATE POINTS ON A ROTATING WHEEL

Imagine that a small Ferris wheel has a radius of 1 decameter (about 33 feet) and that your seat is at a point A when the wheel begins to turn counterclockwise about its center C.



- How does the x-coordinate of your seat change as the wheel turns?
- How does the y-coordinate of your seat change as the wheel turns?

Find the angle of rotation between  $0^\circ$  and  $360^\circ$  that will take the seat from point A to the following points.

- Maximum and minimum distance from the horizontal axis.  $90^\circ$  to maximum  
 $270^\circ$  to minimum

b. Maximum and minimum distance from the vertical axis.  $\text{Max} \rightarrow 0^\circ, 360^\circ$

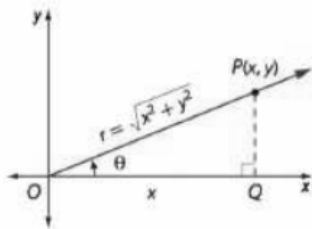
$\text{min} \rightarrow 180^\circ$

c. Points with equal x- and y-coordinates

$45^\circ, 225^\circ$

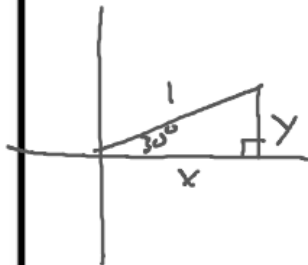
d. Points with opposite x- and y-coordinates

$135^\circ, 315^\circ$



When a circle like that modeling the Ferris wheel is placed on a rectangular coordinate grid with center at the origin  $(0, 0)$ , you can use what you know about geometry and trigonometry to find the x- and y-coordinates of any point on the circle.

Find the coordinates of points that tell the location of the Ferris wheel seat that begins at point  $A(1, 0)$  when the wheel undergoes the following rotations. Record the results on a sketch that shows a circle and the points with their coordinates labels.



$\theta = 30^\circ$   $(.866, .5)$   $\theta = 70^\circ$   $(.342, .939)$   $\theta = 90^\circ$   $(0, 1)$

$\sin 30^\circ = \frac{y}{1}$

$\sin 30^\circ = y$

$\cos 30^\circ = \frac{x}{1}$

$\cos 30^\circ = x$

$\theta = 120^\circ$

$(-.5, .866)$

$\sin 70^\circ = \frac{y}{1}$

$\sin 70^\circ = y = .939$

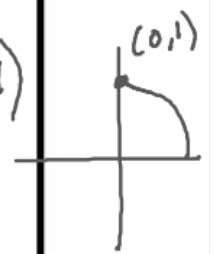
$\cos 70^\circ = \frac{x}{1}$

$\cos 70^\circ = .342$

$\theta = 140^\circ$

$\theta = 180^\circ$

$(-1, 0)$



$$\theta = 220^\circ$$

$$(-.766, -.642)$$

$$\theta = 270^\circ$$

$$(0, -1)$$

$$\theta = 310^\circ$$

$$(.642, -.766)$$

When the Ferris wheel has rotated through an angle of  $40^\circ$ , the seat that started at  $A(1, 0)$  will be at about  $A'(0.77, 0.64)$ . Explain how the symmetry of the circle allows you deduce the location of the seat after rotations of  $140^\circ, 220^\circ, 320^\circ$  and some other angles as well.

Suppose that  $P(x, y)$  is a point on the Ferris wheel model with  $m\angle PCA = \theta$  in degrees.

- What are the coordinates of  $x$  and  $y$ ?
- How will the coordinates values be different if the radius of the circle is  $r$  decameters?

With your calculator set in degree mode, graph the functions  $\cos \theta$  and  $\sin \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ . Compare the patterns in those of graphs to your ideas in first problem.