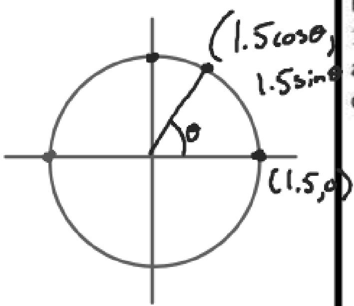
What you will learn about:
Patterns of Periodic Change



If a Ferris wheel has a radius of 1.5 decimeters (15 meters), the function $1.5\cos\theta$ *and* $1.5\sin\theta$ give the x- and y-coordinates after rotation of θ for a seat that starts at the (1.5, 0). Compare the graphs of these new coordinate functions with the graphs of the basic sine and cosine functions.

Find the maximum and minimum points of the graphs of cos θ and 1.5cos θ when:

	max	min
	$\cos 0^\circ = 1$	$\cos 180^\circ = -1$
θ is measured in degrees	$1.5\cos 0^\circ = 1.5$	$1.5\cos 180^\circ = -1.5$
θ is measured in radians	$\cos 0 = 1$	$\cos\pi = -1$
	$1.5\cos 0 = 1.5$	$1.5\cos\pi = -1.5$

Find the θ-axis intercepts of the graphs of cos θ and 1.5cos θ when:

θ is measured in degrees $90^\circ, 270^\circ$

θ is measured in radians $\frac{\pi}{2}$ $\frac{3\pi}{2}$

Find the maximum and minimum points of the graphs of sin θ and 1.5sin θ when:

	max	min
θ is measured in degrees	90°	270°
θ is measured in radians	$\frac{\pi}{2}$	$\frac{3\pi}{2}$

Find the θ-axis intercepts of the graphs of sin θ and 1.5sin θ when:

θ is measured in degrees $0^\circ, 180^\circ, 360^\circ$

θ is measured in radians $0, \pi, 2\pi$

$r=1$ $r=1.5$

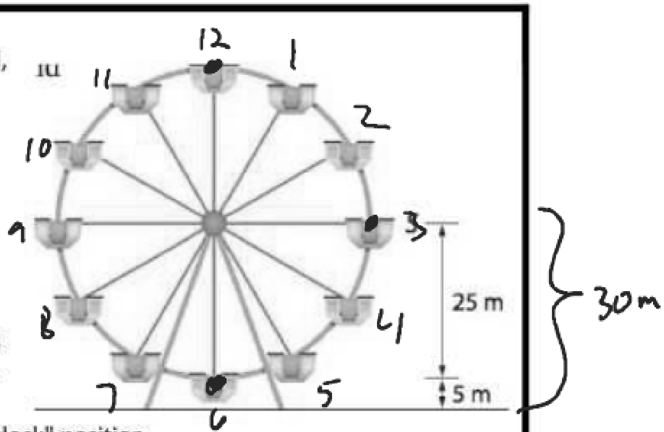
How would the maximum and minimum points on the θ-axis intercept change if the Ferris wheel being modeled had radius ***a*** and the coordinate functions were asin θ and acos θ?

max
$(a,0), (0,a)$
min
$(-a,0)$ $(0,-a)$

$x = a\cos\theta$ $y = a\sin\theta$

radius = a

When riding a Ferris wheel, customers are probably more nervous about their height above ground that their distance from the vertical axis of the wheel. Suppose a large Ferris wheel has a radius of 25 meters, the center of the wheel is located 30 meters above the ground, and the wheel starts in motion when seat S is at the "3 o'clock" position.



Modify the sine function to get a rule h(θ) that gives the height of seat S in meters after rotation of θ. Compare the graph of this height with the graph of sin θ. $h\theta = 25 \sin\theta + 30$

Find the maximum and minimum points on the graph of sin θ and h(θ) when:

	max = 55	min = 5
θ is measured in degrees	90	270
θ is measured in radians	$\frac{\pi}{2}$	$\frac{3\pi}{2}$

Find the θ-axis intercepts on the graphs of sin θ and h(θ)

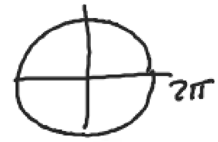
None

$h(\theta) = a\sin\theta + c$

How would the maximum and minimum points and the θ-axis intercept change if the Ferris wheel being modeled had a radius ***a*** and its center was ***c*** meters above the ground? Why is ***c*** > ***a***?

15

12



$\frac{Max - min}{2}$

Amp = Difference Between max and min ÷ 2

Suppose that the height of a Ferris wheel seat changes in a pattern that can be modeled by the function $h(t) = 25 \sin t + 30$, where time is in minutes and height is in meters.

What are the period and amplitude of h(t)? What do those values tell about the motion of the Ferris wheel.

$A = \frac{55-5}{2} = 25$

Amp = 25 → Radius of circle

Per = 2π → 6.28 min

If a seat starts out in the "3 o'clock" position, how long will it take the seat to return to that position? At what times will it revisit that position?

6.28 min

12.56 min

Suppose the height (in meters) of seats on different Ferris wheels changes over time (in minutes) according to the functions give below. For each function:

- Find the height of the seat when the motion of the wheel begins
- Find the amplitude of h(t). Explain what it tells you about motion of the wheel.

$h(t) = a \sin Bx + c$

Per $\frac{2\pi}{B}$

$\frac{2\pi}{\frac{1}{2}}$

$\frac{2\pi}{1} \cdot \frac{2}{3} = \frac{4\pi}{3}$

$h(t) = 15 \sin 0.5t + 17$

Initial height = 17m

Amp = 15 → radius

max = 32m min = 2m

Per = $\frac{2\pi}{.5}$ = 4π = 12.56 min

$h(t) = 24 \cos 2t + 27$

Initial height = 51 min

Amp = 24

max = 51 min 3

Per = $\frac{2\pi}{2}$ = π = 3.14 min

$h(t) = 12 \sin 1.5t + 13$

Initial height = 13

Amp = 12

max = 25 min 1

Per $\frac{2\pi}{B} = \frac{2\pi}{\frac{3}{2}}$

$\frac{4\pi}{3}$ = 4.18 min

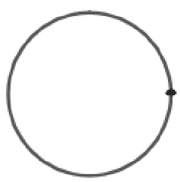
$h(t) = -12 \cos t + 14$

Initial height 2

Amp = 12

max = 26 min = 2

Per $\frac{2\pi}{1}$ = 2π = 6.28 min



Pendulums are among the simplest but most useful examples of periodic motion. Once set in motion, the arm of the pendulum swings left and right on a vertical axis. The angel of displacement from vertical is a periodic function of time that depends on the length of the pendulum and its initial release point.

Suppose that the function $d(t) = 35\cos 2t$ give the displacement from vertical (in degrees) of the tire swing pendulum shown at the right as a function of time (in seconds).

What are the amplitude and period of d(t)? What does each tell you about the motion of the swing?

Amp = 35 Per $\frac{2\pi}{2} = \pi = 3.14$ sec

If the motion of a different swing is modeled by $f(t) = 45\cos \pi t$, what are the amplitude and period of f(t)? What does each tell you about the motion of the swing? Amp = 45 Per $\frac{2\pi}{\pi} = 2$ sec

Why does it make sense to use variation of the circular function $\cos t$ to model pendulum motion?

Start at max value.

What function g(t) would model the motion of a pendulum that is released from a displacement of 18° right of vertical and sings with a frequency of 0.25 cycles per second (a period of 4 seconds)?

At every location on Earth, the number of hours of daylight varies with the season in a predictable way. One convenient way to model that pattern of change is to measure time in days, beginning with spring equinox (about March 21st) as t = 0. With that frame of reference, the number of daylight hours in Boston, Massachusetts is given by $d(t) = 3.5\sin\frac{2\pi}{365}t + 12.5$.

What are the amplitude and period of the d(t)? What do those values tell about the pattern of change in daylight during a year in Boston?