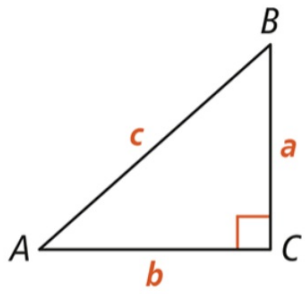
Section 8.1 Pythagorean Theorem and Special Right Triangles

Pythagorean Theorem

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

PROOF: SEE EXAMPLE 1.

If... $\triangle ABC$ is a right triangle.



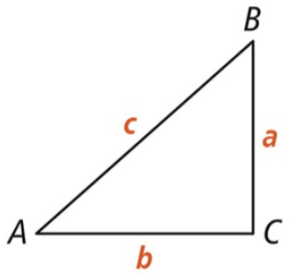
Then... $a^2 + b^2 = c^2$

Converse of the Pythagorean Theorem

If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

PROOF: SEE EXERCISE 17.

If... $a^2 + b^2 = c^2$



Then... $\triangle ABC$ is a right triangle.

Comparing a^2, b^2, and c^2

$a^2+b^2=c^2$ = Rt Δ

$a^2+b^2<c^2$ = Acute Δ

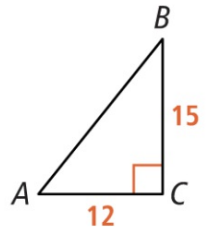
$a^2+b^2>c^2$ = Obtuse Δ

Right

Obtuse

Acute

Use the figure shown. Find *AB*.



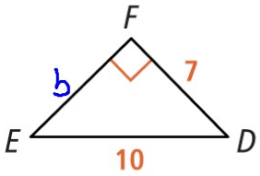
$12^2 + 15^2 = c^2$

$144 + 225 = c^2$

$369 = c^2$

$c = 19.20$

Use the figure shown. Find *EF*.



$7^2 + b^2 = 10^2$

$49 + b^2 = 100$

$b^2 = 51$

$b = 7.14$

Classify the triangle with the given sides as acute, obtuse, or right.

2, 3, 4

$2^2+3^2 \stackrel{?}{=} 4^2$

$4+9=16$

$13<16$

Acute

3, 4, 5

$3^2+4^2=5^2$

$9+16=25$

$25=25$

Right

4, 5, 6

4^2+5^2

$16+25$

$41>36$

Obtuse

1.2, 2.0, 2.5

(1.2, 2.0, 2)

$1.2^2+2^2 \quad 2^2$

$1.44+4 \quad 4$

$5.44>4$

Obtuse

0.6, 0.8, 0.9

$.36+.64 \quad .81$

$1.0>.81$

Obtuse

1, 1, $\sqrt{2}$

$1^2+1^2=(\sqrt{2})^2$

$1+1=2$

$2=2$

Right

1, $\sqrt{2}$, 2

$1^2+(\sqrt{2})^2 \quad 2^2$

$1+2 \quad 4$

$3<4$

Acute

$\sqrt{2}$, 2, $\sqrt{5}$

$(\sqrt{2})^2+2^2 \quad (\sqrt{5})^2$

$2+4 \quad 5$

$6>5$

Obtuse

11, 14, 17

Obtuse

11, 12, 17

Acute

12, 35, 37

Right

12^2+35^2

$144+1225$

1, 3, 6

$1^2+3^2 \quad 6^2$

$1+9 \quad 36$

$10<36$

No $\triangle$

A. To satisfy safety regulations, the distance from the wall to the base of a ladder should be at least one-fourth the length of the ladder. Did Drew set up the ladder correctly?

SOLUTION

$2.5^2 + 9^2 = c^2$

$6.25 + 81 = c^2$

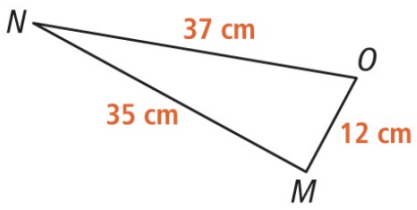
$87.25 = c^2$

$(9.34)(.25)$

2.33

Yes ladder is set up correctly

Is $\triangle MNO$ a right triangle? Explain.



$12^2 + 35^2 \stackrel{?}{=} 37^2$

$144 + 1225 = 1369$

$1369 = 1369$

Yes

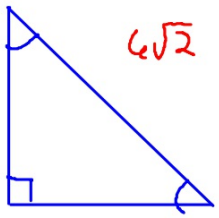
Special Right Triangles

45° - 45° - 90°

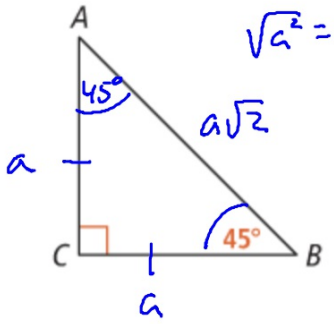
Is there a relationship between the lengths of $\overline{AB}$ and $\overline{AC}$ in $\triangle ABC$? Explain.

SOLUTION

- Legs are same length
- Hypotenuse is Leg times $\sqrt{2}$



$$\frac{6\sqrt{2}}{\sqrt{2}} = 6$$



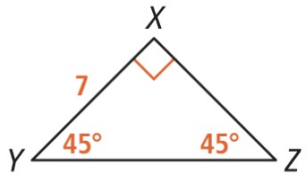
$\sqrt{a^2} = a$

$a^2 + a^2 = c^2$

$\sqrt{2a^2} = \sqrt{c^2}$

$c = a\sqrt{2}$

3. Find the side lengths of the 45°-45°-90° triangle.

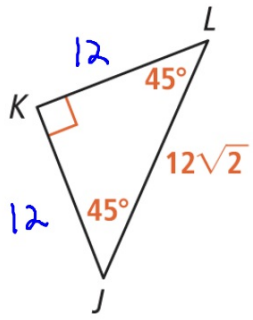


a. What are *XZ* and *YZ*?

$XZ = 7$

$YZ = 7\sqrt{2}$

3. Find the side lengths of the 45°-45°-90° triangle.



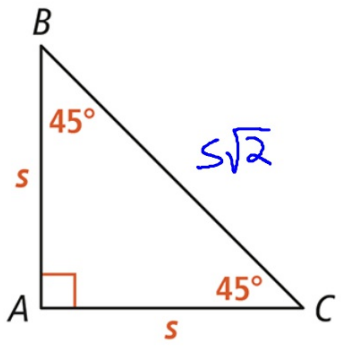
b. What are *JK* and *LK*?

Enter your answer.

45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

If...

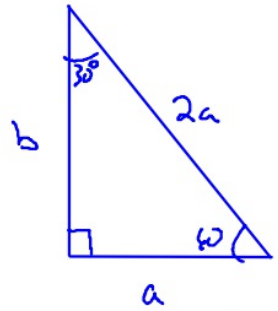


Then... $BC = s\sqrt{2}$

PROOF: SEE EXERCISE 18.

Special Right Triangles

30°- 60° - 90°

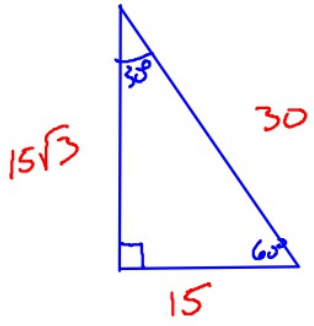


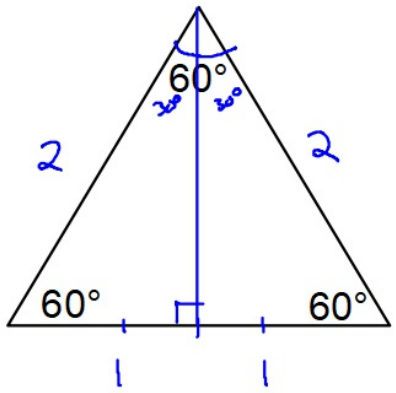
$a^2+b^2=(2a)^2$

$a^2+b^2=4a^2$

$\sqrt{b^2}=\sqrt{3a^2}$

$b=a\sqrt{3}$

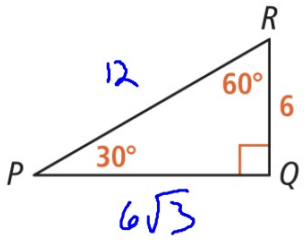




- Short leg is half the hypotenuse
- Long leg is equal to short leg times $\sqrt{3}$

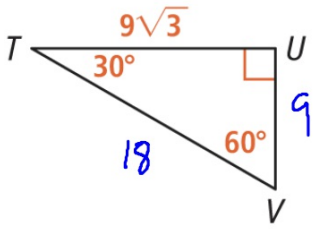
a. What are *PQ* and *PR*?

Enter your answer.



What are *UV* and *TV*?

ıter your answer.

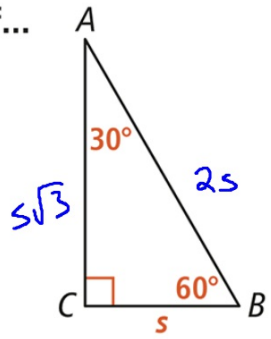


30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the short leg. The length of the long leg is $\sqrt{3}$ times the length of the short leg.

PROOF: SEE EXERCISE 19.

If...



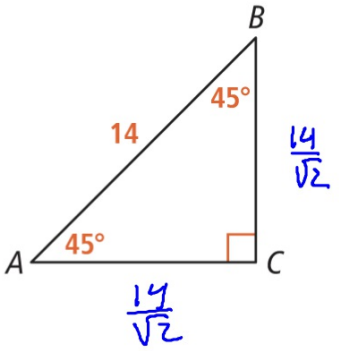
Then... $AC = s\sqrt{3}$, $AB = 2s$

b. What are AC and BC?

Enter your answer.

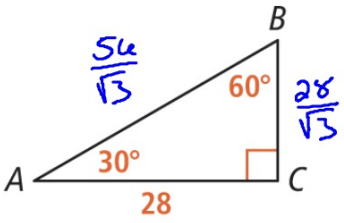
$\frac{14}{\sqrt{2}} = \frac{14}{\sqrt{2}} = 7\sqrt{2}$

$9.89 = 9.89$

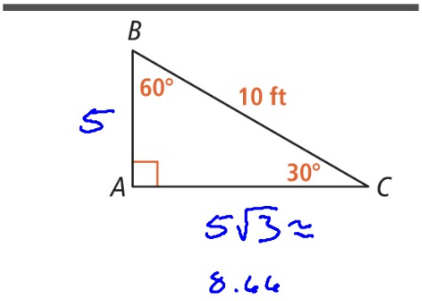


What are AB and BC?

iter your answer.



A. Alejandro needs to make both the horizontal and vertical supports, $\overline{AC}$ and $\overline{AB}$, for a ramp. Is one 12-foot board long enough for both supports? Explain.



SOLUTION

13.66 ft

No 12' board
not enough!