Geometry

Double Reflection over Parallel Lines

COMPOSITE TRANSFORMATIONS

1a) $T_{(-3,5)} \circ R_{y-axis} (\triangle ABC)$

b) $R_{y-axis} \circ T_{(-3,5)} (\triangle ABC)$

c) Did doing the transformations in a different order matter? Explain why?

2a) $R_{x-axis} \circ r_{90^\circ,0} (\triangle ABC)$

b) $r_{90^\circ,0} \circ R_{x-axis} (\triangle ABC)$

c) Did doing the transformations in a different order matter? Explain why?
**DOUBLE REFLECTIONS OVER PARALLEL LINES** – Plot each of the stages of the composite transformation.

3a) \( R_{x=-2} \circ R_{x=-5}(BC) \)  
(Reflection over \( x = -5 \) followed by a reflection over \( x = -2 \))

Circle the resultant transformation from \( BC \) to \( B''C''' \)

- **Rotation**
- **Reflection**
- **Translation**

What is the distance \( BB'' \)? __6__
What is the distance between the parallel lines? __3__

How do these two distances relate to each other? The distance between parallel lines is half the translation distance.

b) \( R_{x=-5} \circ R_{x=-2}(BC) \)  
(Reflection over \( x = -2 \) followed by a reflection over \( x = -5 \))

Circle the resultant transformation from \( BC \) to \( B''C''' \)

- **Rotation**
- **Reflection**
- **Translation**

What is the distance \( BB'' \)? __6__
What is the distance between the parallel lines? __3__

How do these two distances relate to each other?

**CONJECTURE (Take a guess)**

A double reflection over parallel lines seems to produce a ________________ (type of transformation).

What is the relationship between the distance between the parallel lines and the total distance mapped by each point \( (BB'' \ or \ CC'') \)?

The two examples above the only thing that changed was the order that we did the reflection in. What impact did the order have on the result?
DOES THE LOCATION OF THE SHAPE DETERMINE THE DIRECTION IT MOVES?

4a) \( R_{x=2} \circ R_{x=-4} (\triangle ABC) \) (Reflection over \( x = -4 \) followed by a reflection over \( x=2 \))

Circle the resultant transformation from \( \triangle ABC \) to \( \triangle A''B''C'' \)?

Rotation   Reflection   Translation

What is the distance \( CC'' \)? \( 12 \)

What is the distance between the parallel lines? \( 6 \)

How do these two distances relate to each other?

4b) \( R_{x=2} \circ R_{x=-4} (\triangle ABC) \) (Reflection over \( x = -4 \) followed by a reflection over \( x=2 \))

Circle the resultant transformation from \( \triangle ABC \) to \( \triangle A''B''C'' \)?

Rotation   Reflection   Translation

What is the distance \( CC'' \)? \( 12 \)

What is the distance between the parallel lines? \( 6 \)

How do these two distances relate to each other?

CONJECTURE (Take a guess)

Looking back on the two examples, did the starting location of the shape alter the direction of the transformation? No Both transformations finished to the right

What seems to be controlling the direction of the transformation? The order in which the fig are reflected.
5a) $R_{y=5} \circ R_{y=1}(AB)$
(Reflection over $y=1$ followed by a reflection over $y=5$)

Circle the resultant transformation from $AB$ to $A''B''$?

Rotation  Reflection  Translation

What is the distance BB'’?  8
What is the distance between the parallel lines?  4
How do these two distances relate to each other?

What direction did the translation go?  UP  DOWN  RIGHT  LEFT

5b) $R_{y=1} \circ R_{y=5}(AB)$
(Reflection over $y=5$ followed by a reflection over $y=1$)

Circle the resultant transformation from $AB$ to $A''B''$?

Rotation  Reflection  Translation

What is the distance BB’’?  8
What is the distance between the parallel lines?  4
How do these two distances relate to each other?

What direction did the translation go?  UP  DOWN  RIGHT  LEFT
CONJECTURE (Take a guess)

In the two examples, the only thing that changed was the order that we did the reflections. What impact did the order have on the result? How can you determine whether it will go up or down when reflecting over two horizontal lines?

6. **Determine the translation distance and direction from the given information.**
   (Lines that appear to be either horizontal or vertical are.)

   a) $R_n \circ R_m(\triangle ABC)$
      Translation Distance = \(6\)
      Up Down Left Right

   b) $R_n \circ R_m(\triangle ABC)$
      Translation Distance = \(10\)
      Up Down Left Right

   c) $R_n \circ R_m(\triangle ABC)$
      Translation Distance = \(14\)
      Up Down Left Right

   d) $R_m \circ R_n(\triangle ABC)$
      Translation Distance = \(24\)
      Up Down Left Right

   e) $R_m \circ R_n(\triangle ABC)$
      Translation Distance = \(2\)
      Up Down Left Right

   f) $R_m \circ R_n(\triangle ABC)$
      Translation Distance = \(3\)
      Up Down Left Right
7. Summary of relationships found in previous exercises.
   a) Summarize why the order matters for two transformations.

   b) A double reflection over two parallel lines results in a ________________.

   c) The distance of the translation is exactly ____________________ the distance between the parallel lines.

   d) The direction of the translation depends on ________________________________.

8. Complete the following
   a) A reflection over $x = 4$ followed by a reflection over $x = 9$
      results in a translation in the direction of UP DOWN RIGHT LEFT a total distance of ________.

   b) A reflection over $x = -1$ followed by a reflection over $x = -8$
      results in a translation in the direction of UP DOWN RIGHT LEFT a total distance of ________.

   c) A reflection over $y = 7$ followed by a reflection over $y = 3$
      results in a translation in the direction of UP DOWN RIGHT LEFT a total distance of ________.

   d) A reflection over $y = -1$ followed by a reflection over $y = 11$
      results in a translation in the direction of UP DOWN RIGHT LEFT a total distance of ________.